

## Special Relativity: A Centenary Perspective

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### 1 Introduction

A hundred years ago, Einstein laid the foundation for a revolution in our conception of time and space, matter and energy. In his remarkable 1905 paper “On the Electrodynamics of Moving Bodies” [1], and the follow-up note “Does the Inertia of a Body Depend upon its Energy-Content?” [2], he established what we now call special relativity as one of the two pillars on which virtually all of physics of the 20th century would be built (the other pillar being quantum mechanics). The first new theory to be built on this framework was general relativity [3], and the successful measurement of the predicted deflection of light in 1919 made both Einstein the person and relativity the theory internationally famous. The next great theory to incorporate relativity was the Dirac equation

of quantum mechanics; later would come the stunningly successful relativistic theory of quantum electrodynamics.

Strangely, although general relativity had its crucial successes, such as the bending of starlight and the explanation of the advance of Mercury’s perihelion, special relativity was not so fortunate. Indeed, many scholars believe that a lack of direct experimental support for special relativity in the years immediately following 1905 played a role in the decision to award Einstein’s 1921 Nobel Prize, not for relativity, but for one of his other 1905 “miracle” papers, the photoelectric effect, which *did* have direct confirmation in the laboratory.

And although there were experimental tests, such as improved versions of the Michelson-Morley experiment, the Ives-Stilwell experiment, and others, they did not seem to have the same impact as the light-deflection experiment. Still, during the late 1920s and after, special relativity was inexorably accepted by mainstream physicists (apart from those who participated in the anti-Semitic, anti-relativity crusades that arose in Germany and elsewhere in the 1920s, coincident with the rise of Nazism), until it became part of the standard toolkit of every working physicist. Quite the opposite happened to general relativity, which for a time receded to the backwaters of physics, largely because of the perceived absence of further experimental tests or consequences. General relativity would not return to the mainstream until the 1960s.

On the 100th anniversary of special relativity, we see that the theory has been so thoroughly integrated into the fabric of modern physics that its validity is rarely challenged, except by cranks and crackpots. It is ironic then, that during the past several years, a vigorous theoretical and experimental effort has been launched, on an international scale, to find violations of special relativity. The motivation for this effort is not a desire to repudiate Einstein, but to look for evidence of new physics “beyond” Einstein, such as apparent violations of Lorentz invariance that might result from certain models of quantum gravity. So far, special relativity has passed all these new high-precision tests, but the possibility of detecting a signature of quantum gravity, stringiness, or extra dimensions will keep this effort alive for some time to come.

In this paper we endeavor to provide a centenary perspective of special relativity. In Section 2, we discuss special relativity from a historical and pedagogical viewpoint, describing the basic postulates and consequences of special relativity, at a level suitable for non-experts, or for experts who are called upon to teach special relativity to non-experts. In Section 3, we review some of the classic experiments, and discuss the famous “twin paradox” as an example of a frequently misunderstood “consistency” test of the theory. Section 4 discusses special relativity in the broader context of curved spacetime and general relativity, describes how long-range fields interacting with matter can produce “effective” violations of Lorentz invariance and discusses experiments to constrain such violations. In Section 5 we discuss whether gravity itself satisfies a version of Lorentz invariance, and describe the current experimental constraints. In Section 6 we briefly review the most recent extended theoretical frameworks that have been developed to discuss the possible ways of violating Lorentz invariance, as well as some of the ongoing and future experiments to look for such violations. Section 7 presents concluding remarks.

## 2 Fundamentals of special relativity

### 2.1 Einstein’s postulates and insights

Special relativity is based on two postulates that are remarkable for their simplicity, yet whose consequences are far-reaching. They state [1]:

- The laws of physics are the same in any inertial reference frame.
- The speed of light in vacuum is the same as measured by any observer, regardless of the velocity of the inertial reference frame in which the measurement is made.

The first postulate merely adopts the wisdom, handed down from Galileo and Newton, that the laws of mechanics are the same in any inertial frame, and extends it to cover *all* the laws of physics, notably electrodynamics, but also laws yet to be discovered. There is nothing radical or

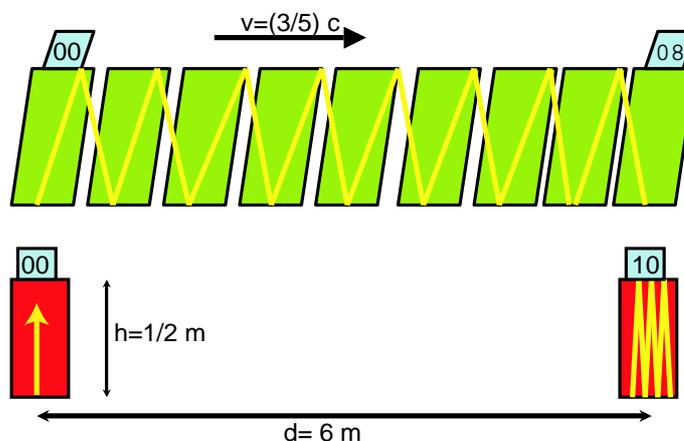


Figure 1: Time dilation of a clock moving at  $v = 3/5c$  between two identical laboratory clocks a distance 6 m apart. The laboratory clocks each tick 10 times during the passage, while the moving clock ticks only 8 times because the light rays travel farther to complete each tick, as seen from the laboratory.

unreasonable about this postulate. It is the second postulate, that the speed of light is the same to all observers, that is usually regarded as radical, yet it is also strangely conservative. Maxwell's equations stated that the speed of light was a fundamental constant, given by  $c = 1/\sqrt{\epsilon_0\mu_0}$ , where  $\epsilon_0$  and  $\mu_0$  are the dielectric permittivity and magnetic permeability of vacuum, two constants that could be measured in the laboratory by performing experiments that had nothing obvious to do with light. That speed  $c$ , now defined to be *exactly* 299,792,458 m/sec, bore no relation to the state of motion of emitter or receiver. Furthermore, there existed a set of transformations, found by Lorentz, under which Maxwell's equations were invariant, with an invariant speed of light.

In addition, Einstein was presumably aware of the Michelson-Morley experiment (although he did not refer to it by name in his 1905 paper) which demonstrated no effect on the speed of light of our motion relative to the so-called "aether" [4]. While the great physicists of the day, such as Lorentz, Poincaré and others were struggling to bring all these facts together by proposing concepts such as "internal time", or postulating and then rejecting "aether drift", Einstein's attitude seems to have been similar to that expressed in the American idiom: "if it walks like a duck and quacks like a duck, it's a duck". If light's speed *seems* to be constant, then perhaps it really *is* a constant, no matter who measures it. Throughout his early career, Einstein demonstrated an extraordinary gift for taking a simple idea at face value and "running" with it; he did this with the speed of light; he did it with Planck's quantum hypothesis and the photoelectric effect, also in 1905.

## 2.2 Time out of joint

An immediate and deep consequence of the second postulate is that time loses its absolute character. First, the rate of time depends on the velocity of the clock. A very simple way to see this is to imagine a thought experiment involving three identical clocks. Each clock consists of a chamber of length  $h$  with a perfect mirror at each end. A light ray bounces back and forth between the mirrors, recording one "tick" each time it hits the bottom mirror. In the rest frame of each clock the speed of light is  $c$  (by the second postulate), so the duration of each "tick" is  $2h/c$  according to observers on each clock. Two of the clocks are at rest in a laboratory, a distance  $d$  apart along the  $x$ -axis, arranged so that the light rays move in the  $y$ -direction. The two clocks have been synchronized using a light flash from a lamp midway between them. The third clock moves with velocity  $v$  in

the  $x$ -direction (Fig. 1). As it passes each of the laboratory clocks in turn, its own reading and the reading on the adjacent laboratory clock are taken and later compared. The time difference between the readings on the two laboratory clocks is clearly  $d/v$  or  $(d/v)/(2h/c)$  ticks. But from the point of view of the laboratory, the light ray on the moving clock moves in a saw-tooth manner as the mirrors move, with the distance along the hypotenuse of each tooth given by  $l = \sqrt{h^2 + (vt)^2}$  where  $t$  is the time taken as seen from the lab. But at the speed of light, this time is given by  $l/c$ , so the duration of a “tick” on the moving clock from the lab viewpoint is given by  $(2h/c)\gamma$ , where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . Thus the number of ticks on the moving clock between its encounters with the lab clocks is  $(d/v)/(2h/c) \times \sqrt{1 - v^2/c^2}$ . If we define “proper time”  $\Delta\tau$  as the time elapsed on a single clock between two events at its own location, and  $\Delta t$  as the time difference measured by the two separated laboratory clocks, then

$$\Delta\tau = \Delta t \sqrt{1 - v^2/c^2}. \quad (1)$$

This is the time dilation: the time elapsed between two events along the path of a single moving clock is less than that measured by a *pair* of synchronized clocks located at the two events. The asymmetry is critical: A clock can only make time readings along its own world line, thus *two* synchronized clocks are required in the laboratory, in order to make comparisons with readings on the moving clock.

While this time dilation was already recognized at some level by Lorentz and others as a consequence of the Lorentz transformations, they were unable or unwilling to recognize its true meaning, because they remained wedded to the Newtonian view of an absolute time. Einstein, possibly because of his early contact with the machinery and equipment of his father’s factories, was able to view time operationally: time is what clocks measure. If one thinks of a clock as any device that performs some precisely repetitive activity governed fundamentally by the laws of physics, then it becomes obvious that time in the moving frame really does tick more slowly than in the lab. And this is not some abstract, internal time, this is time measured by our mirror clock, by an atomic clock, by a biological clock, by a human heartbeat, all of which are governed by the laws of physics, which are the same in every inertial frame. From any conceivable observable viewpoint this *is* time.

In the thought experiment above we remarked that the laboratory clocks were synchronized. This seemingly obvious and innocuous statement also has deep consequences, because, as Einstein realized, if the speed of light is the same for all observers, then synchronization is relative. Consider two observers on the ground who synchronize their clocks by setting them to read the same when a light flash from a point midway between them is received. Now consider observers on a train moving by (who have previously synchronized their own clocks using the same method on the train). The light flash emitted by the lamp on the ground has speed  $c$  in both directions as seen from the train (second postulate), therefore the forward moving flash will encounter the forward ground clock (which is moving toward the lamp as seen from the train) *before* the backward moving flash encounters the rear ground clock (which is receding). The events of reception of the light flash by the two ground clocks are simultaneous in the ground frame, but are *not* simultaneous in the train’s frame. Again, this was embodied mathematically in the Lorentz transformation, but it was Einstein who inferred this truth about time: events simultaneous in one frame, are not automatically simultaneous in a moving frame.

Much has been written about why Einstein was able to arrive at this new view of time, while his contemporaries, including great men like Lorentz and Poincaré, were not. Henri Poincaré is a case in point. By 1904 Poincaré understood almost everything there was to understand about relativity. In 1904 he journeyed to St. Louis to speak at the scientific congress associated with the World’s Fair, on the newly relocated campus of my own institution, Washington University. In reading Poincaré’s paper “The Principles of Mathematical Physics” [5], one senses that he is so close to having special relativity that he can almost taste it. Yet he could not take the final leap to the new understanding of time. This is ironic, because as Peter Galison has written [6], Poincaré was one of the world’s leaders in the understanding of clock synchronization, having served on French and international agencies and committees charged with establishing the world-wide conventions for time-synchronization and time transfer that were needed for transportation, navigation and

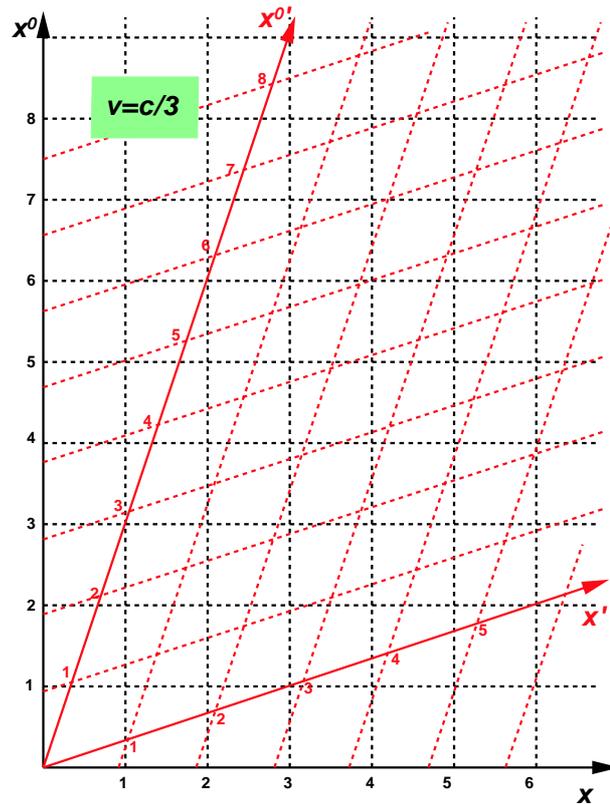


Figure 2: Spacetime diagram showing a laboratory frame and a frame moving at  $v = c/3$ .

telegraphy. Surely Poincaré would have understood our example of the moving train, yet it seems that he could not go beyond viewing it as merely conventional. To Einstein, it reflected what clocks measure, and therefore reflected the true nature of time.

### 2.3 Spacetime and Lorentz invariance

If the speed of light is the same to all observers, then time and space can be put on a similar footing initially by measuring time in units of distance, so that  $t$  in meters stands for  $ct$ , and corresponds to the time it takes for light to travel one meter (3.336 nanoseconds). We will call this time in distance units the coordinate  $x^0$ . One can then describe space and time together on a spacetime diagram, with points representing “events”, “worldlines” representing the trajectories of particles through space and time and so on.

A train moving with speed  $v$ , with the caboose passing the origin at  $x^0 = 0$  has the collection of world lines shown in Fig. 2 (one for each car in the train), each with slope  $1/v$ . The line passing through the origin is called the  $x^{0'}$  axis, just as in Galilean relativity. By carefully considering how clocks on the train would be synchronized, either using a master lamp as in the example above, or by using round-trip signals (often called Einstein synchronization), it is easy to show that the collection of events on the train that are simultaneous with the origin lie along the  $x'$ -axis shown, with slope  $v$ . Later “lines of simultaneity” on the train are also shown. Figure 2 makes it clear how all observers can agree on the speed of light. A light ray emanating from the origin of Fig. 2 follows a  $45^\circ$  line, or a line that bisects the  $x$  and  $x^0$  axes. But that line also bisects the  $x'$  and  $x^{0'}$  axes, thus observers on the train will also find speed  $c$  for that ray.

These considerations establish only the slopes of the lines, however. They do not tell us where, for example, to mark 1 meter on the  $x'$ -axis. To resolve this, we return to our simple moving clock example, and notice that, while the time difference and spatial difference between

the events describing one “tick” of the moving clock are given by  $\Delta t' = 2h/c$  and  $\Delta x' = 0$  in its own frame, and by the different values  $\Delta t = \gamma(2h/c)$  and  $\Delta x = v\Delta t = v\gamma(2h/c)$  in the lab frame, the quantity  $\Delta s^2 \equiv -c^2\Delta t^2 + \Delta x^2$  is the same for the tick, whether calculated in the clock’s frame or in the lab frame. This is the “invariant interval”, given for general infinitesimal displacements by

$$\begin{aligned} ds^2 &= -c^2 dt^2 + dx^2 + dy^2 + dz^2 \\ &= -(dx^0)^2 + dx^2 + dy^2 + dz^2 \\ &= \eta_{\mu\nu} dx^\mu dx^\nu, \end{aligned} \quad (2)$$

where  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  is the Minkowski metric, Greek indices run over four spacetime values, and we use the Einstein convention of summing over repeated indices. If one then asks, what linear transformations from one inertial frame to a moving inertial frame will leave this interval invariant in form, or equivalently will leave the Minkowski metric invariant, the answer is the Lorentz transformations: for a boost in the  $x$ -direction, they are given by

$$\begin{aligned} (x^0)' &= \gamma(x^0 - vx), \\ x' &= \gamma(x - vx^0). \end{aligned} \quad (3)$$

For a general boost with velocity  $v^i$ , they are given by  $x^{\alpha'} = \Lambda_{\beta}^{\alpha'} x^\beta$ , where

$$\Lambda_0^{0'} = \gamma, \quad \Lambda_i^{0'} = \Lambda_0^{i'} = -\gamma v^i, \quad \Lambda_j^{i'} = \delta_j^i + (\gamma - 1)v^i v^j / v^2. \quad (4)$$

This is called Lorentz invariance of the interval (or metric). The form of the interval is also invariant under ordinary rotations, and under displacements such as  $x^\alpha \rightarrow x^\alpha + a^\alpha$ . Collectively this larger 10 parameter invariance is called Poincaré invariance. The Lorentz transformations then allow one to establish the scale of the axes of the moving frame, as shown in Fig. 2 for the case  $v = c/3$ .

These are the same transformations, of course, as those found to leave Maxwell’s equations invariant. Einstein’s first postulate, that the laws of physics should be the same in every inertial frame, therefore places a stringent constraint on the design of any future fundamental laws, namely that they should be Lorentz invariant, at least when viewed from an inertial frame. This constraint has guided the great advances in fundamental theory of the 20th century, such as relativistic quantum mechanics and the Dirac equations, quantum electrodynamics, quantum chromodynamics, superstring theory, not to mention general relativity.

## 2.4 Special relativistic dynamics

By considering the acceleration of a charged particle in an electromagnetic field and imposing the principle of relativity [1], Einstein concluded that the equations of dynamics would have to be modified. Further, in another characteristic example of his ability to use a simple thought experiment to derive profound consequences, Einstein established the equivalence between mass and energy [2]. He considered the simple situation of a particle emitting an equal amount of electromagnetic radiation in opposite directions. He then considered the same situation from the viewpoint of a moving inertial frame. By imposing conservation of energy in both frames, and using the transformation laws for electromagnetic radiation, he concluded, working in the low-velocity limit, that the difference in kinetic energy of the particle before and after the emission, as seen in the moving frame, had to be given by  $\frac{1}{2}Ev^2/c^2$ , where  $E$  is the energy of the emitted light. But since kinetic energy in this limit is given by  $\frac{1}{2}mv^2$ , then the mass of the particle must have changed by  $E/c^2$  during the emission of energy  $E$ .

What emerged from these considerations was a new relativistic dynamics. One must replace the Newtonian formulation of  $\mathbf{F} = m\mathbf{a}$  with a relativistically correct formulation  $\vec{F} = d\vec{p}/d\tau$ , where the force  $\vec{F}$  is now a four-vector,  $\vec{p}$  is the four-momentum, given for a particle of rest mass  $m_0$  by  $\vec{p} = m_0\vec{u}$ , where the four-velocity  $\vec{u}$  has components  $u^\alpha = dx^\alpha/d\tau$ , and where  $d\tau = ds/c$  denotes proper time along the particle’s worldline. If the force is provided by electromagnetic fields, then  $F^\nu = (e/c)u_\mu F^{\mu\nu}$ , where  $e$  is the charge of the particle, and  $F^{\mu\nu}$  is the antisymmetric Faraday

tensor, whose components in a given inertial frame may be identified as  $F_{i0} = E_i$ ,  $F_{ij} = \epsilon_{ijk}B_k$ , where  $E^i$  and  $B^i$  are the normal electric and magnetic fields. This dynamics, along with Maxwell's equations, can be derived from the action

$$I = -\sum_a m_{0a}c \int (-\eta_{\mu\nu}u_a^\mu u_a^\nu)^{1/2} d\tau + \sum_a \frac{e_a}{c} \int A_\mu(x_a^\nu) dx_a^\mu - \frac{1}{16\pi} \int \sqrt{-\eta} \eta^{\mu\alpha} \eta^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} d^4x, \quad (5)$$

where  $u_a^\mu$  is the four-velocity of the particle,  $A_\mu(x^\nu)$  is the electromagnetic four-vector potential, and  $F_{\mu\nu} \equiv \partial A_\nu / \partial x^\mu - \partial A_\mu / \partial x^\nu$ . In ordinary variables, in a given inertial frame, the action takes the form

$$I = -\sum_a m_{0a}c^2 \int (1 - v_a^2/c^2)^{1/2} dt + \sum_a e_a \int (-\Phi + \mathbf{A} \cdot \mathbf{v}_a/c) dt + \frac{1}{8\pi} \int (E^2 - c^2 B^2) d^3x dt, \quad (6)$$

where  $\Phi = -A_0$ ,  $\mathbf{E} = -\nabla\Phi - \dot{\mathbf{A}}/c$ , and  $\mathbf{B} = \nabla \times \mathbf{A}$ .

### 3 Classic tests of special relativity

#### 3.1 The Michelson-Morley experiment

From today's perspective the null result of the 1887 Michelson-Morley aether-drift experiment marked the beginning of the end for the Newtonian notions of absolute space and time. Yet it took almost 20 years for the new view of spacetime to be realized. The experiment was beautiful in its simplicity, and should have been a "slam dunk" for conventional 19th century physics. If the speed of light is a fundamental constant, then it must take this value in some preferred frame, presumably that of a luminiferous aether, which would be at rest with respect to the universe, and which would provide the medium that every one thought was necessary for the propagation of light. For any observer moving relative to the aether, the speed of light would be formed by subtracting the velocity vector of the observer from that of the light ray. In one of the interferometers that Michelson had pioneered for measuring the speed of light itself, the speed of light up and down an arm that was parallel to our motion through the aether would be  $c + v$  and  $c - v$ , while the speed along an arm perpendicular to our motion would be  $\sqrt{c^2 + v^2}$ . For an equal-arm interferometer of length  $h$ , the difference in round trip travel time along the two arms would then be, to first order in  $(v/c)^2$ ,  $\Delta T = (h/c)(v/c)^2$ . This would be reflected in a change in the interference pattern of the recombined beams, that would shift as the apparatus was rotated, thereby interchanging the roles of the two arms.

But instead of the predicted shift, Michelson and Morley found no effect, and placed an upper limit on a shift 40 times smaller than the shift predicted [4], and later experiments only improved the bounds (see [7] for a review up to 1955). Attempts to explain this by arguing that the aether was "dragged" by the Earth proved to be untenable. Lorentz wrote to Lord Rayleigh in 1892, "I am totally at a loss to clear away this contradiction . . . Can there be some point in the theory of Mr. Michelson's experiment which has been overlooked?" [7]. Lorentz and FitzGerald attempted to resolve the problem by proposing that the interferometer arms parallel to the motion through the aether were shortened by the factor  $\sqrt{1 - v^2/c^2}$ , but could not suggest what this meant [8, 9].

Special relativity resolved the Michelson-Morley experiment instantly. In the rest frame of the experiment, the speed of light is the same, irrespective of the instrument's motion relative to the universe, so the experiment should automatically give a null result. Indeed, the aether now becomes completely irrelevant. Alternatively, from the point of view of a frame at rest relative to the universe, careful consideration of how length is measured in special relativity showed that the interferometer arm moving parallel to its length must be shortened by the precise Lorentz-FitzGerald factor. The null experimental result could be derived from either frame of reference.

In placing the Michelson-Morley (MM) experiment in a modern context, it is useful to view it not as an interferometer experiment, but as a clock anisotropy experiment. Each arm of the interferometer can be thought of as a clock just like the clocks used in Sec. 2.2 above. The fundamental question then becomes, is the rate of a clock independent of its orientation relative to its motion through the universe? Most modern incarnations of the MM experiment are clock anisotropy experiments. For example, MM experiments using lasers [10, 11] compare two laser resonant cavities by beating their frequencies against each other as one or both rotate relative to the universe.

One can invent a way to parametrize the MM experiment so as to quantify how the null result could be violated, that turns out to be useful in more general contexts. Suppose that, working in the rest frame of the universe (we may discard the aether, but the rest-frame of the universe, as reflected by the rest frame of the cosmic background radiation, has a well defined meaning), the speed of light is  $c$ . But suppose that the Lorentz-FitzGerald contraction of the parallel arm is given by the factor  $\sqrt{1 - v^2/c_0^2}$ , where  $c_0$  is a different speed (measured in the universe rest frame), that is connected with whatever dynamics determines the structure of the walls of the cavity that forms our clock. Then it is easy to show that, while the time for one tick of the clock perpendicular to the motion is given by  $(2h/c)(1/\sqrt{1 - v^2/c^2})$ , the time for one tick of the parallel clock is  $(2h/c)[\sqrt{1 - v^2/c_0^2}/(1 - v^2/c^2)]$ . To first order in  $(v/c)^2$ , the differential clock time is given by  $(h/c)(v/c_0)^2\delta$ , where  $\delta = (c_0/c)^2 - 1$ .

If Lorentz invariance holds, then the electrodynamics that governs the solids that form the cavity must involve the same  $c$  as that which governs the propagation of light, hence  $c_0 = c$ ,  $\delta = 0$  and we recover the null prediction for the MM experiment. Below we will discuss classes of theories that involve curved spacetime plus certain kinds of long-range fields, in which this no longer holds. Figure 4 shows selected bounds on  $\delta$  that were achieved in the original MM experiment, and in later experiments of the MM type by Joos and a 1979 test using laser technology by Brillet and Hall [11]. In that Figure, units are chosen so that  $c_0 = 1$ .

### 3.2 Invariance of $c$

Several classic experiments have been performed to verify that the speed of light is independent of the speed of the emitter. If the speed of light were given by  $\mathbf{c} + k\mathbf{v}$ , where  $\mathbf{v}$  is the velocity of the emitter, and  $k$  is a parameter to be measured or bounded, then orbits of binary star systems would appear to have an anomalous eccentricity unexplainable by normal Newtonian gravity. However, at optical wavelengths, this test is not unambiguous because light is absorbed and reemitted by the intervening interstellar medium, thus losing the memory of the speed of the source, a phenomenon known as extinction. But at X-ray wavelengths, the path length of extinction is tens of kiloparsecs, so nearby X-ray binary sources in our galaxy may be used to test the velocity dependence of light. Using data on pulsed 70 keV X-ray binary systems, Her S-1, Cen X-3 and SMC X-1, Brecher [12] obtained a bound  $|k| < 2 \times 10^{-9}$ , for typical orbital velocities  $v/c \sim 10^{-3}$ .

At the other extreme, a 1964 experiment at CERN used ultrarelativistic particles as the source of light. Neutral pions were produced by the collisions of 20 GeV protons on stationary nucleons in the proton synchrotron. With energies larger than 6 GeV, the pions had  $v/c \geq 0.99975$ . Photons produced by the decay  $\pi^0 \rightarrow \gamma_1 + \gamma_2$  were collimated and timed over a 30 meter long flight path. Because the protons in the synchrotron were pulsed, the speed of the photons could be measured by measuring the arrival times of their pulses as a function of the varying location of the detector along the flight path. The result for the speed was  $2.9977 \pm 0.0004 \times 10^8$  m/sec, in agreement with the laboratory value [13]. This experiment thus set a bound  $|k| < 10^{-4}$  for  $v \approx c$ .

### 3.3 Time dilation

The observational evidence for time dilation is overwhelming. Ives and Stilwell [14] measured the frequency shifts of radiation emitted in the forward and backward direction by moving ions of  $\text{H}_2$  and  $\text{H}_3$  molecules. The first-order Doppler shift cancels from the sum of the forward and backward shifts, leaving only the second-order time-dilation effect, which was found to agree with theory. (Ironically, Ives was a die-hard opponent of special relativity.)

A classic experiment performed by Rossi and Hall [15] showed that the lifetime of  $\mu$ -mesons was prolonged by the standard factor  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . Muons are created in the upper atmosphere when cosmic ray protons collide with nuclei of air, producing pions, which decay to muons. With a rest half-life of  $2.2 \times 10^{-6}$  s, a muon travelling near the speed of light should travel only 2/3 of a kilometer on average before decaying to a harmless electron or positron and two neutrinos. Yet muons are the primary component of cosmic radiation detected at sea level. But with time dilation and a typical speed of  $v/c \sim 0.994$ , their lives as seen from Earth are prolonged by a factor of nine, easily enough for them to reach sea level. Rossi and Hall measured the distribution of muons as a function of altitude and also measured their energies, and confirmed the time dilation formula. In fact, since collisions between cosmic ray muons and DNA molecules are a non-negligible source of natural genetic mutations, one could even argue that special relativity plays a role in evolution!

In an experiment performed in 1966 at CERN, muons produced by collisions at one of the targets in the accelerator were deflected by magnets so that they would move on circular paths in a “storage ring”. Their speeds were 99.7 percent of the velocity of light, and the observed twelve-fold increase in their lifetimes agreed with the prediction with 2 percent accuracy [16].

### 3.4 Lorentz invariance and quantum mechanics

The integration of Lorentz invariance into quantum mechanics has provided a string of successes for special relativity. The first was the discovery of the Dirac equation, the relativistic generalization of Schrödinger quantum mechanics, with its prediction of anti-particles and elementary particle spin. Another was the development of relativistic quantum field theory. QFT naturally embodies the Pauli exclusion principle, by requiring that the creation and annihilation operators of spinor fields satisfy anticommutation relations in order to obey Lorentz invariance. Since the Pauli exclusion principle explains the occupation of atomic energy levels by electrons, one could argue, with but a hint of chauvinism, that special relativity explains Chemistry! The modern incarnations of QFT, such as Quantum Electrodynamics, Electroweak Theory, Quantum Chromodynamics all have Lorentz invariance as foundations. However, until recently, the experimental successes of such theories have not been used to attempt to quantify how well Lorentz invariance holds. We will return to this subject in Sec. 6.

### 3.5 Consistency tests of special relativity

Over the years, special relativity has been subjected to a series of tests, not of its experimental predictions, but of its very logic. Many of its predictions, such as the slowing of time on moving clocks, were deemed to be so strange, so beyond normal experience, that there had to be something wrong with the theory. The idea was to find “paradoxes”, simple situations where the theory could be shown to be logically inconsistent.

Of course, there are no paradoxes! To be sure, the idea of time dilation may be hard to understand or to swallow, but there is absolutely nothing paradoxical about it.

The most popular of these is, of course, the twin paradox. In his 1905 paper, Einstein himself presents the situation clearly [1]: “If one of two synchronous clocks at  $A$  is moved in a closed curve with constant velocity until it returns to  $A$ , the journey lasting  $t$  seconds, then by the clock which has remained at rest the travelled clock on its arrival at  $A$  will be  $\frac{1}{2}tv^2/c^2$  seconds slow.”

The more modern versions of the story go something like this: On New Year’s Day 3000, an astronaut ( $A$ ) sets out from Earth at speed  $0.6c$  and travels to the nearest interstellar Space Station, Clinton-1, which is 3 light-years away as measured in the Earth frame of reference (Fig. 3). Having reached Clinton-1, she immediately turns around and returns to Earth at the same speed, arriving home on New Year’s Day 3010, by Earth time. The astronaut has a twin brother ( $B$ ), who remains on Earth.

From the point of view of Earth’s inertial frame, astronaut  $A$ ’s clock runs slow, with her proper time elapsed on the outbound journey being given by Eq. (1), amounting to 4 years, compared with 5 years on Earth. The times elapsed on the return journey are the same (the total proper time elapsed during the accelerated motion needed for the turnaround can be made as small as

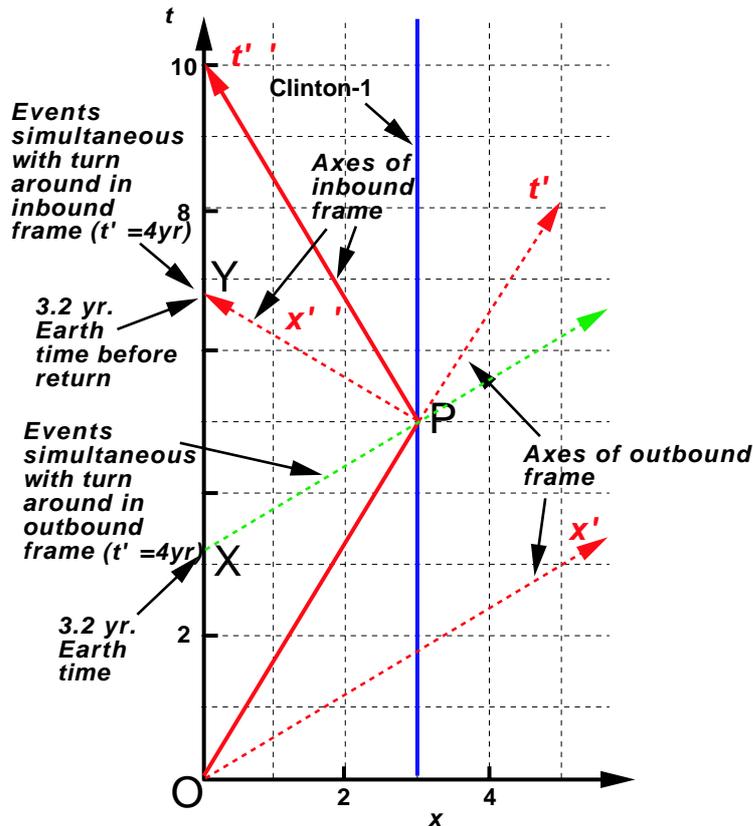


Figure 3: Twin Paradox as seen from traveller's viewpoint

one likes applying large accelerations for a short time). Astronaut *A* returns having aged 8 years, compared to the 10 years aging of her twin brother.

The “paradox” is then stated as follows: from the astronaut *A*'s point of view, Earth's clocks run slow, so *A* should return *older* than her brother, not *younger*. Since this is a logical contradiction, relativity is untenable.

The flaw in the “paradox” is the failure to comprehend what is meant by “*A* sees Earth's clock run slow”. *A* cannot compare her clock with Earth's clock because she is nowhere near Earth except at the start of the journey. Instead, an inertial frame moving outbound with *A*'s velocity must be created, with a set of observers carrying clocks synchronized with hers. The readings on Earth's clock can only be read by one of these observers who happens to be passing the Earth at that moment of time. But because of the relativity of simultaneity, the event in this outbound frame that is simultaneous with *A*'s turnaround event *P* is *not* the 5-year mark on Earth, but is event *X* on Fig. 3, which is at Earth year 3003.2. So observers in *A*'s outbound frame do agree that Earth's clock has run slow compared to hers, 3.2 years compared to 4 years. But while *A* decelerates and accelerates for the return journey, that outbound inertial frame continues flying off at  $0.6c$  forever, and *A* must pick up a new inertial frame inbound at  $0.6c$ . In *that* frame, the event that is simultaneous with the turnaround is at event *Y*, Earth year 3006.8, 3.2 years before the return. Again, observers in the inbound inertial frame agree that Earth's clock runs slow during the return journey, 3.2 years, compared to *A*'s 4 years. But the analysis using the two inertial frames has failed to account for the 3.6 years between events *X* and *Y*.

This is not a paradox, it's merely sloppy accounting (perhaps the twin paradox should be renamed the Enron of Relativity). With a knowledge of the relativity of simultaneity, astronaut *A* could easily conclude that the gap between the two lines of simultaneity corresponding to her turnaround is 3.6 years; alternatively she could consult observers in an infinite sequence of inertial

frames corresponding to all the velocities of her spacecraft from  $v$  to  $-v$  and add up all the infinitesimal increments of Earth's clock as read by these observers, and account for the 3.6 missing years. Either way, she reaches the unambiguous conclusion that she ages a total of 8 years, while her twin ages 10 years.

It is sometimes claimed that the resolution of the twin paradox must ultimately involve general relativity, because the traveller accelerates, and acceleration is equivalent to gravitation. As the discussion above shows, acceleration plays no role in the analysis, other than to provide the asymmetry whereby the traveller must occupy more than one inertial frame, while the home-bound twin occupies a single inertial frame throughout. The relativity of simultaneity is the key, not gravity.

In fact, the relativity of simultaneity is the key to resolving essentially all of the “paradoxes” that have been devised to test the logical structure of special relativity, such as the “pole in the barn” paradox (a rapidly moving pole is short enough to fit inside a barn, at least momentarily, from the barn's point of view, but can't possibly fit from the pole's point of view), the “space-war paradox”, “the jumping frog paradox” and others. For discussion of these and many other paradoxes, see [17].

## 4 Special relativity and curved spacetime

Special relativity and general relativity are often viewed as being independent. One reason for this apparent division is that Einstein presented special relativity 100 years ago in 1905, while general relativity was not published in its final form until 1916. Another reason is that the two parts of the theory have very different realms of applicability: special relativity mainly in the world of microscopic physics, and general relativity in the world of astrophysics and cosmology.

But in fact, the theory of relativity is a single, all-encompassing theory of space-time, gravity and mechanics. Special relativity is actually an approximation to curved space-time that is valid in sufficiently small regions of space-time (called “local freely falling frames”), much as small regions on the surface of an apple are approximately flat, even though the overall surface is curved. Special relativity can therefore be used whenever the scale of the phenomena being studied is small compared with the scale on which the curvature of space-time (i.e. gravity) begins to be noticed. For most applications in atomic or nuclear physics, this approximation is so accurate that special relativity can be assumed to be exact.

Historically, however, Einstein's journey from special to general relativity was tortuous and difficult. It began in 1907 with what he has called “the happiest thought” of his life. According to numerous experiments, all laboratory-sized bodies fall with the same acceleration, regardless of their mass, composition or structure, in a given external gravitational field. Einstein was probably aware of experiments performed by Eötvös around the turn of the 20th century [18], that demonstrated this “universality of free fall” to parts in  $10^9$ . The modern bounds are at the level of parts in  $10^{13}$  [19].

From this simple fact, Einstein noticed that if an observer were to ride in an elevator falling freely in a gravitational field, then all bodies inside the elevator would move uniformly in straight lines as if gravity had vanished. Conversely, in an accelerated elevator in free space, where there is no gravity, the bodies would fall with the same acceleration because of their inertia, just as if there were a gravitational field.

Einstein's great insight was to postulate that this “vanishing” of gravity in free fall or its “presence” in an accelerating frame applied not only to mechanical motion but to all the laws of physics, such as electromagnetism. Thus, in an accelerating frame, a light ray moving horizontally would be seen to be deflected downward, and a ray moving upward or downward would have its frequency shifted [20, 21].

For the next 8 years, Einstein looked for a theory that would embody this principle of equivalence, be compatible with Lorentz invariance in the absence of gravity, and reflect his goals of elegance and simplicity, succeeding finally in the fall of 1915 [3].

#### 4.1 Einstein's equivalence principle

Our modern viewpoint of the foundations of general relativity is based on an extension and embellishment of Einstein's principle of equivalence. Much of this viewpoint can be traced back to Robert Dicke, who contributed crucial ideas about the foundations of gravitation theory between 1960 and 1965. These ideas were summarized in his influential Les Houches lectures of 1964 [22] and resulted in what has come to be called the Einstein equivalence principle (EEP), which states that

- test bodies fall with the same acceleration independently of their internal structure or composition (universality of free fall, also called the weak equivalence principle, or WEP);
- the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed (local Lorentz invariance, or LLI)
- the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed (local position invariance, or LPI).

The Einstein equivalence principle is the heart of gravitational theory, for it is possible to argue convincingly that if EEP is valid, then gravitation must be described by “metric theories of gravity”, which state that (i) spacetime is endowed with a symmetric metric, (ii) the trajectories of freely falling bodies are geodesics of that metric, and (iii) in local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity. For further discussion, see [23].

One way to see that spacetime cannot be flat is the following. Consider two freely-falling frames on opposite sides of the Earth. According to the Einstein equivalence principle, space-time is Minkowskian in each frame, but because the frames are accelerating toward each other, the two space-times cannot be extended and meshed into a single Minkowskian space-time. In the presence of gravity, space-time is flat locally but curved globally.

#### 4.2 Metric theories of gravity

The simplest way to incorporate the Einstein equivalence principle mathematically into the special relativistic dynamics of particles and fields is to replace the Minkowski metric in the action of Eq. (5) with the curved-spacetime metric  $g_{\mu\nu}$ , and to replace ordinary derivatives with covariant derivatives, yielding the action

$$\begin{aligned}
 I = & - \sum_a m_{0a} c \int (-g_{\mu\nu} u_a^\mu u_a^\nu)^{1/2} d\tau + \sum_a \frac{e_a}{c} \int A_\mu(x_a^\nu) dx_a^\mu \\
 & - \frac{1}{16\pi} \int \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} d^4x, \tag{7}
 \end{aligned}$$

where  $d\tau = ds/c$ , with  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ . The only way that “gravity” enters is via the metric  $g_{\mu\nu}$ . Any theory whose equations for matter can be cast into this form is called a metric theory.

As a result, the non-gravitational interactions couple *only* to the spacetime metric  $g_{\mu\nu}$ , which locally has the Minkowski form  $\eta_{\mu\nu}$  of special relativity. Because this local interaction is only with  $\eta_{\mu\nu}$ , local non-gravitational physics is immune from the influence of distant matter, apart from tidal effects. Local physics is Lorentz invariant (because  $\eta_{\mu\nu}$  is) and position invariant (because  $\eta_{\mu\nu}$  is constant in space and time).

General relativity is a metric theory of gravity, but so are many others, including the Brans-Dicke theory. In this sense, superstring theory is not metric, because there is a residual coupling of external, gravitation-like fields, to matter. Theories in which varying non-gravitational constants are associated with dynamical fields that couple to matter directly are also not metric theories.

### 4.3 Effective violations of local Lorentz invariance

How could violations of LLI arise? From the viewpoint of field theory, violations would generically be caused by other long-range fields in addition to  $g_{\mu\nu}$  which also couple to matter, such as scalar, vector and tensor fields. Theories that have this property are called non-metric theories. A simple example of such a theory is one in which the matter action is given by

$$I = -\sum_a m_{0a} c \int (-g_{\mu\nu} u_a^\mu u_a^\nu)^{1/2} d\tau + \sum_a \frac{e_a}{c} \int A_\mu(x_a^\nu) dx_a^\mu - \frac{1}{16\pi} \int \sqrt{-h} h^{\mu\alpha} h^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} d^4x, \quad (8)$$

where  $h_{\mu\nu}$  is a second, second-rank tensor field. Locally, one can always find coordinates (local freely-falling frame) in which  $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ , but in general  $h_{\mu\nu} \not\rightarrow \eta_{\mu\nu}$ ; instead  $h_{\mu\nu} \rightarrow (h_0)_{\mu\nu}$ , where  $(h_0)_{\mu\nu}$  is a tensor whose values are determined by the cosmology or nearby mass distribution. In the rest frame of the distant matter distribution,  $(h_0)_{\mu\nu}$  will have specific values, and there is no reason *a priori* why those should correspond to the Minkowski metric (unless  $h_{\mu\nu}$  were identical to  $g_{\mu\nu}$  in the first place, in which case one would have a metric theory). Also, in a frame moving with respect to the distant sources of  $h_{\mu\nu}$ , the local values of  $(h_0)_{\mu\nu}$  will depend on the velocity of the frame, thereby producing *effective* violations of Lorentz invariance in electrodynamics.

A number of explicit theoretical frameworks were developed between 1973 and 1990 to treat non-metric theories of this general type. They include the “ $TH\epsilon\mu$ ” framework of Lightman and Lee [24], the  $\chi - g$  framework of Ni [25], the  $c^2$  framework of Haugan and coworkers [26, 27], and the extended  $TH\epsilon\mu$  framework of Vucetich and colleagues [28].

In the  $c^2$  framework, one assumes a class of non-metric theories in which the particle and interaction parts of the action Eq. (8) can be put into the local special relativistic form, using units in which the limiting speed of neutral test particles is unity, and in which the sole effect of any non-metric field coupling to electrodynamics is to alter the effective speed of light. The result is the action

$$I = -\sum_a m_{0a} \int (1 - v_a^2)^{1/2} dt + \sum_a e_a \int (-\Phi + \mathbf{A} \cdot \mathbf{v}_a) dt + \frac{1}{8\pi} \int (E^2 - c^2 B^2) d^3x dt. \quad (9)$$

Because the action is explicitly non-Lorentz invariant if  $c \neq 1$ , it must be defined in a preferred universal rest frame, presumably that of the 3K microwave background. In this frame, the value of  $c^2$  is determined by the cosmological values of the non-metric field. Even if the non-metric field coupling to electrodynamics is a tensor field, the homogeneity and isotropy of the background cosmology in the preferred frame is likely to collapse its effects to that of the single parameter  $c^2$ . Detailed calculations of a variety of experimental situations show that those “preferred-frame” effects depend on the magnitude of the velocity through the preferred frame ( $\sim 350$  km/sec), and on the parameter  $\delta \equiv c^{-2} - 1$ . In any metric theory or theory with local Lorentz invariance,  $\delta = 0$ .

One can then set observable upper bounds on  $\delta$  using a variety of experiments. In the Michelson-Morley experiment, by considering the behavior of amorphous solids in the dynamics above, one can show that the length of the “parallel” clock is shortened by the factor  $\sqrt{1 - v^2}$ ; in our units, the speed  $c_0$  of Sec. 3.1 is unity. Thus the MM experiment sets the bound  $\delta < 10^{-3}$ .

Better bounds on  $\delta$  have been set by other “standard” tests of special relativity, such as descendants of the Michelson-Morley experiment [4, 7, 11], a test of time-dilation using radionuclides on centrifuges [29], tests of the relativistic Doppler shift formula using two-photon absorption (TPA) [30], and a test of the isotropy of the speed of light using one-way propagation of light between hydrogen maser atomic clocks at the Jet Propulsion Laboratory (JPL) [31].

Very stringent bounds  $|\delta| < 10^{-21}$  have been set by “mass isotropy” experiments of a kind pioneered by Hughes and Drever [32, 33]. The idea is simple: in a frame moving relative to the preferred frame, the non-Lorentz-invariant electromagnetic action of Eq. (9) becomes anisotropic,

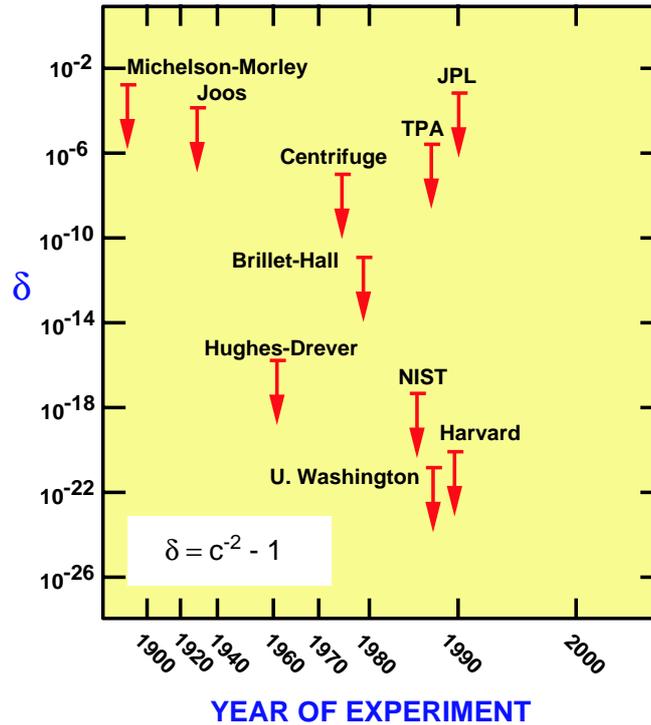


Figure 4: Bounds on violations of local Lorentz invariance

dependent on the direction of the velocity  $\mathbf{V}$ . Those anisotropies then are reflected in the energy levels of electromagnetically bound atoms and nuclei (for nuclei, we consider only the electromagnetic contributions). For example, the three sublevels of an  $l = 1$  atomic or nuclear wavefunction in an otherwise spherically symmetric atom can be split in energy, because the anisotropic perturbations arising from the electromagnetic action affect the energy of each substate differently. One can study such energy anisotropies by first splitting the sublevels slightly using a magnetic field, and then monitoring the resulting Zeeman splitting as the rotation of the Earth causes the laboratory  $\mathbf{B}$ -field (and hence the quantization axis) to rotate relative to  $\mathbf{V}$ , causing the relative energies of the sublevels to vary among themselves diurnally. Using nuclear magnetic resonance techniques, the original Hughes-Drever experiments placed a bound of about  $10^{-16}$  eV on such variations. This is about  $10^{-22}$  of the electromagnetic energy of the nuclei used. Since the magnitude of the predicted effect depends on the product  $V^2\delta$ , and  $V^2 \approx 10^{-6}$ , one obtains the bound  $|\delta| < 10^{-16}$ . Energy anisotropy experiments were improved dramatically in the 1980s using laser-cooled trapped atoms and ions [34, 35, 36]. This technique made it possible to reduce the broadening of resonance lines caused by collisions, leading to improved bounds on  $\delta$  shown in Figure 4 (experiments labelled NIST, U. Washington and Harvard, respectively).

## 5 Is gravity Lorentz invariant?

The strong equivalence principle (SEP) is a generalization of EEP which states that in local “freely-falling” frames that are large enough to include gravitating systems (such as planets, stars, a Cavendish experiment, a binary system, *etc.*), yet that are small enough to ignore tidal gravitational effects from surrounding matter, local *gravitational* physics should be independent of the velocity of the frame and of its location in space and time. Also *all* bodies, including those bound by their own self-gravity, should fall with the same acceleration. General relativity satisfies SEP, whereas most other metric theories do not (*eg.* the Brans-Dicke theory).

It is straightforward to see how a gravitational theory could violate SEP [37]. Most alternative

metric theories of gravity introduce auxiliary fields which couple to the metric (in a metric theory they can't couple to matter), and the boundary values of these auxiliary fields (determined either by cosmology or by distant matter) can act back on the local gravitational dynamics. The effects can include variations in time and space of the locally measured effective Newtonian gravitational constant  $G$  (preferred-location effects), as well as effects resulting from the motion of the frame relative to a preferred cosmic reference frame (preferred-frame effects). Theories with auxiliary scalar fields, such as the Brans-Dicke theory and its generalizations, generically cause temporal and spatial variations in  $G$ , but respect the ‘‘Lorentz invariance’’ of gravity, *i.e.* produce no preferred-frame effects. The reason is that a scalar field is invariant under boosts. On the other hand, theories with auxiliary vector or tensor fields can cause preferred-frame effects, in addition to temporal and spatial variations in local gravitational physics. For example, a timelike, long-range vector field singles out a preferred universal rest frame, one in which the field has no spatial components; if this field is generated by a cosmic distribution of matter, it is natural to assume that this special frame is the mean rest frame of that matter. A number of such ‘‘vector-tensor’’ metric theories of gravity have been devised [37, 38, 39]; see [23] for a review.

General relativity embodies SEP because it contains only one gravitational field  $g_{\mu\nu}$ . Far from a local gravitating system, this metric can always be transformed to the Minkowski form  $\eta_{\mu\nu}$  (modulo tidal effects of distant matter and  $1/r$  contributions from the far field of the local system), a form that is constant and Lorentz invariant, and thus that does not lead to preferred-frame or preferred-location effects.

The theoretical framework most convenient for discussing SEP effects is the parametrized post-Newtonian (PPN) formalism [40, 41, 23], which treats the weak-field, slow-motion limit of metric theories of gravity. This limit is appropriate for discussing the dynamics of the solar system and for many stellar systems, except for those containing compact objects such as neutron stars. If one focuses attention on theories of gravity whose field equations are derivable from an invariant action principle (Lagrangian-based theories), the generic post-Newtonian limit is characterized by the values of five PPN parameters,  $\gamma$ ,  $\beta$ ,  $\xi$ ,  $\alpha_1$  and  $\alpha_2$ . Two in particular,  $\alpha_1$  and  $\alpha_2$ , measure the existence of preferred-frame effects. If SEP is valid,  $\alpha_1 = \alpha_2 = \xi = 4\beta - \gamma - 3 = 0$ , as in general relativity. In scalar-tensor theories,  $\alpha_1 = \alpha_2 = \xi = 0$ , but  $4\beta - \gamma - 3 = 1/(2 + \omega)$ , where  $\omega$  is the ‘‘coupling parameter’’ of the scalar-tensor theory. In Rosen's bimetric theory,  $\alpha_2 = c_0/c_1 - 1$ ,  $\alpha_1 = \xi = 4\beta - \gamma - 3 = 0$ , where  $c_0$  and  $c_1$  are the cosmologically induced values of the temporal and spatial diagonal components of a flat background tensor field, evaluated in a cosmic rest frame in which the physical metric has the Minkowski form far from the local system.

Within the PPN formalism the variations in the locally measured Newtonian gravitational constant  $G_{\text{local}}$  can be calculated explicitly: viewed as the coupling constant in the gravitational force between two point masses at a given separation, it is given by

$$G_{\text{local}} = 1 - (4\beta - \gamma - 3 - 3\xi)U_{\text{ext}} - \frac{1}{2}(\alpha_1 - \alpha_2)V^2 - \frac{1}{2}\alpha_2(\mathbf{V} \cdot \mathbf{e})^2 + \xi U_{\text{ext}}(\mathbf{N} \cdot \mathbf{e})^2, \quad (10)$$

where  $U_{\text{ext}}$  is the potential of an external mass in the direction  $\mathbf{N}$ ,  $\mathbf{V}$  is the velocity of the experiment relative to the preferred frame,  $\mathbf{e}$  is the orientation of the two masses and units have been chosen so that  $G_{\text{local}} = 1$  in the preferred frame far from local matter sources. Thus  $G_{\text{local}}$  can vary in magnitude with variations in  $U_{\text{ext}}$  and  $V^2$ , and can also be anisotropic, that is can vary with the orientation of the two bodies. Other SEP-violating effects include planetary orbital perturbations and precessions of planetary and solar spin axes. A variety of observations have placed the bounds

$$|\alpha_1| < 10^{-4}, \quad |\alpha_2| < 4 \times 10^{-7}. \quad (11)$$

See [23, 42] for further details about tests of preferred-frame effects in gravity.

## 6 Tests of local Lorentz invariance at the centenary

### 6.1 Frameworks for Lorentz symmetry violations

During the past decade there has been a major renewal of interest in developing new ways to test Lorentz symmetry, using laboratory experiments and astrophysical observations. Part of the moti-

vation for this comes from quantum gravity. Quantum gravity asserts that there is a fundamental length scale given by the Planck length,  $L_p = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$  cm, but since length is not an invariant quantity (Lorentz-FitzGerald contraction), then there could be a violation of Lorentz invariance at some level in quantum gravity. In brane world scenarios, while physics may be locally Lorentz invariant in the higher dimensional world, the confinement of the interactions of normal physics to our four-dimensional “brane” could induce apparent Lorentz violating effects. And in models such as string theory, the presence of additional scalar, vector and tensor long-range fields that couple to matter of the standard model could induce effective violations of Lorentz symmetry, as we discussed in Sec. 4.3. These and other ideas have motivated a serious reconsideration of how to test Lorentz invariance with better precision and in new ways.

Kostalecky and collaborators developed a useful and elegant framework for discussing violations of Lorentz symmetry in the context of the standard model of particle physics [43, 44, 45]. Called the Standard Model Extension (SME), it takes the standard  $SU(3) \times SU(2) \times U(1)$  field theory of particle physics, and modifies the terms in the action by inserting a variety of tensorial quantities in the quark, lepton, Higgs, and gauge boson sectors that could explicitly violate LLI. SME extends the earlier classical frameworks ( $TH\epsilon\mu$ ,  $c^2$ ,  $\chi - g$ ) to quantum field theory and particle physics. The modified terms split naturally into those that are odd under CPT (i.e. that violate CPT) and terms that are even under CPT. The result is a rich and complex framework, with many parameters to be analysed and tested by experiment. Such details are beyond the scope of this paper; for a review of SME and other frameworks, the reader is referred to the recent article by Mattingly [46].

Here we confine our attention to the electromagnetic sector, in order to link the SME with the  $c^2$  framework discussed above. In the SME, the Lagrangian for a scalar particle  $\phi$  with charge  $e$  interacting with electrodynamics takes the form

$$\begin{aligned} \mathcal{L} = & [\eta^{\mu\nu} + (k_\phi)^{\mu\nu}](D_\mu\phi)^\dagger D_\nu\phi - m^2\phi^\dagger\phi \\ & - \frac{1}{4}[\eta^{\mu\alpha}\eta^{\nu\beta} + (k_F)^{\mu\nu\alpha\beta}]F_{\mu\nu}F_{\alpha\beta}, \end{aligned} \quad (12)$$

where  $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$ , and where  $(k_\phi)^{\mu\nu}$  is a real symmetric trace-free tensor, and  $(k_F)^{\mu\nu\alpha\beta}$  is a tensor with the symmetries of the Riemann tensor, and with vanishing double trace. It has 19 independent components. There could also be a CPT-odd term in  $\mathcal{L}$  of the form  $(k_A)^\mu\epsilon_{\mu\nu\alpha\beta}A^\nu F^{\alpha\beta}$ , but because of a variety of pre-existing theoretical and experimental constraints, it is generally set to zero.

The tensor  $(k_F)^{\mu\alpha\nu\beta}$  can be decomposed into “electric”, “magnetic” and “odd-parity” components, by defining

$$\begin{aligned} (\kappa_{DE})^{jk} &= -2(k_F)^{0j0k}, \\ (\kappa_{HB})^{jk} &= \frac{1}{2}\epsilon^{j pq}\epsilon^{k rs}(k_F)^{pqrs}, \\ (\kappa_{DB})^{kj} &= -(k_{HE})^{jk} = \epsilon^{j pq}(k_F)^{0k pq}. \end{aligned} \quad (13)$$

In many applications it is useful to use the further decomposition

$$\begin{aligned} \tilde{\kappa}_{tr} &= \frac{1}{3}(\kappa_{DE})^{jj}, \\ (\tilde{\kappa}_{e+})^{jk} &= \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk}, \\ (\tilde{\kappa}_{e-})^{jk} &= \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \frac{1}{3}\delta^{jk}(\kappa_{DE})^{ii}, \\ (\tilde{\kappa}_{o+})^{jk} &= \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk}, \\ (\tilde{\kappa}_{o-})^{jk} &= \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk}. \end{aligned} \quad (14)$$

The first expression is a single number, the next three are symmetric trace-free matrices, and the final is an antisymmetric matrix, accounting thereby for the 19 components of the original tensor  $(k_F)^{\mu\alpha\nu\beta}$ .

In the rest frame of the universe, these tensors have some form that is established by the global nature of the solutions of the overarching theory being used. In a frame that is moving relative to the universe, the tensors will have components that depend on the velocity of the frame, and on the orientation of the frame relative to that velocity.

In the case where the theory is rotationally symmetric in the preferred frame, the tensors  $(k_\phi)^{\mu\nu}$  and  $(k_F)^{\mu\nu\alpha\beta}$  can be expressed in the form

$$\begin{aligned}(k_\phi)^{\mu\nu} &= \tilde{\kappa}_\phi(u^\mu u^\nu + \frac{1}{4}\eta^{\mu\nu}), \\ (k_F)^{\mu\nu\alpha\beta} &= \tilde{\kappa}_{tr}(4u^{[\mu}\eta^{\nu][\alpha}u^{\beta]} - \eta^{\mu[\alpha}\eta^{\beta]\nu}),\end{aligned}\tag{15}$$

where  $[\ ]$  around indices denote antisymmetrization, and where  $u^\mu$  is the four-velocity of an observer at rest in the preferred frame. With this assumption, all the tensorial quantities in Eq. (14) vanish in the preferred frame, and, after suitable rescalings of coordinates and fields, the action (12) can be put into the form of the  $c^2$  framework, with

$$c = \left(\frac{1 - \frac{3}{4}\tilde{\kappa}_\phi}{1 + \frac{1}{4}\tilde{\kappa}_\phi}\right)^{1/2} \left(\frac{1 - \tilde{\kappa}_{tr}}{1 + \tilde{\kappa}_{tr}}\right)^{1/2}.\tag{16}$$

Another class of frameworks for considering Lorentz invariance violations is kinematical. They involve modifying the relationship between energy  $E$  and momentum  $p$  for each particle species. Assuming that rotational symmetry in the preferred frame is maintained, then one adopts a parametrized dispersion relation of the form

$$E^2 = m^2 + p^2 + E_{Pl}f^{(1)}|p| + f^{(2)}p^2 + \frac{f^{(3)}}{E_{Pl}}|p|^3 + \dots,\tag{17}$$

where  $E_{Pl}$  is the Planck energy. Frameworks like these are useful for discussing effects that might be relics of quantum gravity, and for discussing particle physics and high-energy astrophysics experiments.

## 6.2 Modern searches for Lorentz symmetry violation

A variety of modern ‘‘clock isotropy’’ experiments have been carried out to bound the electromagnetic parameters of the SME framework. For example, comparing the frequency of electromagnetic cavity oscillators of various configurations with atomic clocks as a function of the orientation of the laboratory has placed bounds on the coefficients of the tensors  $\tilde{\kappa}_{e-}$  and  $\tilde{\kappa}_{o+}$  at the levels of  $10^{-15}$  and  $10^{-11}$ , respectively [46]. Direct comparisons between atomic clocks based on different nuclear species place bounds on SME parameters in the neutron and proton sectors, depending on the nature of the transitions involved. The bounds achieved range from  $10^{-27}$  to  $10^{-32}$  GeV [46].

Astrophysical observations have also been used to bound Lorentz violations. For example, if photons satisfy the Lorentz violating dispersion relation (17), then the speed of light  $v_\gamma = \partial E/\partial p$  would be given by

$$v_\gamma = 1 + \frac{(n-1)f_\gamma^{(n)}E^{n-2}}{2E_{Pl}^{n-2}}.\tag{18}$$

By bounding the difference in arrival time of high-energy photons from a burst source at large distances, one could bound contributions to the dispersion for  $n > 2$ . The best limit,  $|f^{(3)}| < 128$  comes from observations of 1 and 2 TeV gamma rays from the blazar Markarian 421 [47].

Other testable effects of Lorentz invariance violation include threshold effects in particle reactions, birefringence in photon propagation through empty space, gravitational Cerenkov radiation, and neutrino oscillations. Mattingly [46] gives a thorough and up-to-date review of both the theoretical frameworks and the experimental results.

## 7 Concluding remarks

At the centenary of special relativity, I can think of no better tribute to the impact and influence of Einstein's relativistic contributions than to cite how they now affect daily life. This unique confluence of abstract theory, high precision technology and everyday applications involves the Global Positioning System (GPS). This navigation system, based on a constellation of 24 satellites carrying atomic clocks, uses precise time transfer to provide accurate absolute positioning anywhere on Earth to 15 meters, differential or relative positioning to the level of centimeters, and time transfer to a precision of 50 nanoseconds. It relies on clocks that are stable, run at the same or well calibrated rates, and are synchronized. However, the difference in rate between GPS satellite clocks and ground clocks caused by the special relativistic time dilation is around -7,000 ns per day, while the difference caused by the gravitational redshift is around 46,000 ns per day. The net effect is that the satellite clocks tick faster than ground clocks by around 39,000 ns per day. Consequently, general relativity *must* be taken into account in order to achieve the 50 ns time transfer accuracy required for 15 m navigation. In addition, the satellite clocks must be synchronized with respect to a fictitious clock on the Earth's rotation axis, in order to avoid the inevitable inconsistency in synchronizing clocks around a closed path in a rotating frame (called the Sagnac effect). For a detailed discussion of relativity in GPS, see [48]; for a popular essay on the subject, see [49]. GPS is a spectacular example of the unexpected and unintended benefits of basic research. While Einstein often used trains to illustrate principles and consequences of relativity, one can now find practical, everyday consequences of relativity in trains, planes and automobiles.

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