Séminaire Poincaré 1 (2004) 23 – 44

String Theory: Past, Present, and Future

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Abstract. String theory was originally developed in an attempt to describe the strong nuclear force. However, it turned out to work better as a unified theory of gravity and the other forces. This manuscript gives a pedagogical introduction to the basic mathematical structure of string theory that led to this conclusion. It also describes the extension to include fermions, which results in the inclusion of supersymmetry and the construction of superstring theory. Following that, it gives a more qualitative discussion of recent developments as well as some of the most important unsolved problems.

1 Introduction

String theory arose in the late 1960s in an attempt to understand the strong nuclear force. This is the force that is responsible for holding protons and neutrons together inside the nucleus of an atom. In the 1970s it was also recognized to be the force that is responsible for holding quarks together inside the protons and neutrons, as well as various other (unstable) strongly interacting particles. A theory based on one-dimensional extended structures, called strings, rather than point-like particles, can account qualitatively for various features of the strong nuclear force and the strongly interacting particles (called hadrons). The basic idea in the string description of the strong interactions is that the various specific particles correspond to specific oscillation modes (or quantum states) of the string. This proposal gives a very satisfying unified picture in that it postulates a single fundamental object (namely, the string) that explains the myriad of different observed hadrons.

This program was not fully successful. Moreover, in the early 1970s another theory of the strong nuclear force – called quantum chromodynamics (or QCD) – was developed. As a result of this, as well as various technical problems in the string theory approach, string theory fell out of favor. The modern viewpoint is that this program made good sense, but that it failed because the correct string theory that gives an alternative (or "dual") description of QCD was not identified. In fact, such a string theory is still not known. The specific string theories that were considered at the time had problems, which we will describe later. However, as will also be discussed in more detail, they were suitable for a completely different and seemingly more ambitious purpose: a quantum theory that unifies the description of gravity and the other fundamental forces.

Even though string theory [1, 2] is not yet fully formulated, and we cannot yet give a detailed description of how the standard model of elementary particles should emerge at low energies, there are some general features of the theory that can be identified. These are features that seem to be quite generic irrespective of how various details are resolved. The first, and perhaps most important, is that general relativity is necessarily incorporated in the theory. It gets modified at very short distances/high energies but at ordinary distances and energies it is present in exactly the form proposed by Einstein. This is significant, because it is arising within the framework of a consistent quantum theory. Ordinary quantum field theory does not allow gravity to exist; string theory requires it! The second general fact is that Yang–Mills gauge theories of the sort that comprise the standard model naturally arise in string theory. We do not understand why the specific $SU(3) \times SU(2) \times U(1)$ gauge theory of the standard model should be preferred, but theories of this general type do arise naturally at ordinary energies. The third general feature of string theory is supersymmetry. The mathematical consistency of string theory depends crucially on supersymmetry, and it is very hard to find consistent solutions (or quantum vacua) that do not preserve at least a portion of this supersymmetry. This prediction of string theory differs from the other two (general relativity and gauge theories) in that it really is a prediction. It is a generic feature of string theory that has not yet been discovered experimentally.

In conventional quantum field theory the elementary particles are mathematical points, whereas in perturbative string theory the fundamental objects are one-dimensional loops (of zero thickness). Strings have a characteristic length scale, which can be estimated by dimensional analysis. Since string theory is a relativistic quantum theory that includes gravity it must involve the fundamental constants c (the speed of light), $\hbar$ (Planck's constant divided by 2π), and G (Newton's gravitational constant). From these one can form a length, known as the Planck length

$$\ell_{\rm p} = \left(\frac{\hbar G}{c^3}\right)^{3/2} = 1.6 \times 10^{-33}\,{\rm cm}. \tag{1.1}$$

Similarly, the Planck mass is[1]

$$m_{\rm p} = \left(\frac{\hbar c}{G}\right)^{1/2} = 1.2 \times 10^{19}\,{\rm GeV}/c^2. \tag{1.2}$$

Experiments at energies far below the Planck energy cannot resolve distances as short as the Planck length. Thus, at such energies, strings can be accurately approximated by point particles. From the viewpoint of someone who believes in string theory, this explains why quantum field theory has been so successful.

As a string evolves in time it sweeps out a two-dimensional surface in spacetime, which is called the world sheet of the string. This is the string counterpart of the world line for a point particle. In quantum field theory, analyzed in perturbation theory, contributions to amplitudes are associated to Feynman diagrams, which depict possible configurations of world lines. In particular, interactions correspond to junctions of world lines. Similarly, string theory perturbation theory involves string world sheets of various topologies. A particularly significant fact is that these world sheets are generically smooth. The existence of interaction is a consequence of world-sheet topology rather than a local singularity on the world sheet. This difference from point-particle theories has two important implications. First, in string theory the structure of interactions is uniquely determined by the free theory. There are no arbitrary interactions to be chosen. Second, the ultraviolet divergences of point-particle theories can be traced to the fact that interactions are associated to world-line junctions at specific spacetime points. Because the string world sheet is smooth, string theory amplitudes have no ultraviolet divergences.

2 Basic String Theory

2.1 World-Volume Actions

In order to set the stage for strings, let me start with a quick review of the world-line description of a relativistic point particle. A point particle sweeps out a trajectory (or world line) in spacetime. This can be described by functions $x^\mu(\tau)$ that describe how the world line, parameterized by τ, is embedded in the spacetime, whose coordinates are denoted x^μ. For simplicity, let us assume that the spacetime is flat Minkowski space with a Lorentz invariant line element is given by

$$ds^2 = -\eta_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - dx^i dx^i. \tag{2.1}$$

In units $\hbar = c = 1$, the action for a particle of mass m is given by

$$S = -m \int ds. \tag{2.2}$$

This could be generalized to a curved spacetime by replacing $\eta_{\mu\nu}$ by a metric $g_{\mu\nu}(x)$, but we will not do so here. This action is stationary for geodesics, which are spacetime trajectories whose

[1] A GeV is 10^9 electron volts and a TeV is 10^{12} electron volts.

invariant length is extremal. As a result, standard relativistic kinematics of point particles follows from the Euler-Lagrange equations of this action.

In terms of the embedding functions, $x^\mu(t)$, the action can be rewritten in the form

$$S = -m \int d\tau \sqrt{-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}, \tag{2.3}$$

where dots represent τ derivatives. An important property of this action is invariance under local reparametrizations. This is a kind of gauge invariance, whose meaning is that the form of S is unchanged under an arbitrary reparametrization of the world line $\tau \to \tau(\tilde{\tau})$. This reparametrization invariance is a one-dimensional analog of the four-dimensional general coordinate invariance of general relativity. Mathematicians refer to this kind of symmetry as diffeomorphism invariance.

We can now generalize the analysis of the massive point particle to a p-brane (an object with p spatial dimensions) of tension T_p. The action in this case involves the invariant $(p+1)$-dimensional world-volume and is given by

$$S_p = -T_p \int d\mu_{p+1}, \tag{2.4}$$

where the invariant volume element is

$$d\mu_{p+1} = \sqrt{-\det(-\eta_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu)}\, d^{p+1}\sigma. \tag{2.5}$$

Here the embedding of the p-brane into d-dimensional spacetime is given by functions $x^\mu(\sigma^\alpha)$. The index $\alpha = 0, \ldots, p$ labels the $p+1$ coordinates σ^α of the p-brane world-volume and the index $\mu = 0, \ldots, d-1$ labels the d coordinates x^μ of the d-dimensional spacetime. We have defined

$$\partial_\alpha x^\mu = \frac{\partial x^\mu}{\partial \sigma^\alpha}. \tag{2.6}$$

The determinant operation acts on the $(p+1) \times (p+1)$ matrix whose rows and columns are labeled by α and β. The tension T_p is interpreted as the mass per unit volume of the p-brane. For a 0-brane, it is just the mass. S_p is reparametrization invariant.

Let us now specialize to the case of a string, which has $p = 1$. Evaluating the determinant gives

$$S[x] = -T \int d\sigma d\tau \sqrt{\dot{x}^2 x'^2 - (\dot{x} \cdot x')^2}, \tag{2.7}$$

where we have defined $\sigma^0 = \tau$, $\sigma^1 = \sigma$, and

$$\dot{x}^\mu = \frac{\partial x^\mu}{\partial \tau}, \quad x'^\mu = \frac{\partial x^\mu}{\partial \sigma}. \tag{2.8}$$

This action is called the Nambu–Goto action [3, 4]. It is proportional to the invariant area of the world-sheet, which is extremal for a classical trajectory. This is the string analog of a geodesic.

The Nambu–Goto action is equivalent to the action

$$S[x,h] = -\frac{T}{2} \int d^2\sigma \sqrt{-h} h^{\alpha\beta} \eta_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu, \tag{2.9}$$

where $h_{\alpha\beta}(\sigma,\tau)$ is the world-sheet metric, $h = \det h_{\alpha\beta}$, and $h^{\alpha\beta}$ is the inverse of $h_{\alpha\beta}$. The Euler–Lagrange equation obtained by varying $h^{\alpha\beta}$ gives the vanishing of the two-dimensional stress-energy tensor

$$T_{\alpha\beta} = \partial_\alpha x \cdot \partial_\beta x - \frac{1}{2} h_{\alpha\beta} h^{\gamma\delta} \partial_\gamma x \cdot \partial_\delta x = 0. \tag{2.10}$$

This equation can be used to eliminate the world-sheet metric from the action, and when this is done one recovers the Nambu–Goto action.

In addition to reparametrization invariance, the action $S[x,h]$ has another local symmetry, conformal invariance. Specifically, it is invariant under the transformation

$$\begin{aligned} h_{\alpha\beta} &\to \Lambda(\sigma,\tau) h_{\alpha\beta} \\ x^\mu &\to x^\mu. \end{aligned} \tag{2.11}$$

This local symmetry is special to the $p = 1$ case (strings).

The two reparametrization invariance symmetries of $S[x, h]$ allow us to choose a gauge in which the three functions $h_{\alpha\beta}$ (this is a symmetric 2×2 matrix) are expressed in terms of just one function. A convenient choice is the "conformally flat gauge"

$$h_{\alpha\beta} = \eta_{\alpha\beta} e^{\phi(\sigma,\tau)}. \tag{2.12}$$

Here, $\eta_{\alpha\beta}$ denoted the two-dimensional Minkowski metric of a flat world sheet. However, because of the factor $e^{\phi}, h_{\alpha\beta}$ is only "conformally flat." Classically, substitution of this gauge choice into $S[x, h]$ leaves the gauge-fixed action

$$S = -\frac{T}{2} \int d^2\sigma \eta^{\alpha\beta} \partial_\alpha x \cdot \partial_\beta x. \tag{2.13}$$

Quantum mechanically, the story is more subtle. Instead of eliminating h via its classical field equations, one should perform a Feynman path integral, using standard machinery to deal with the local symmetries and gauge fixing. When this is done correctly, one finds that in general ϕ does not decouple from the answer. Only for the special case $d = 26$ does the quantum analysis reproduce the formula we have given based on classical reasoning. Otherwise, there are correction terms whose presence can be traced to a conformal anomaly (i.e., a quantum-mechanical breakdown of the conformal invariance) [5].

The gauge-fixed action is quadratic in the x's. Mathematically, it is the same as a theory of d free scalar fields in two dimensions. The equations of motion obtained by varying x^μ are simply free two-dimensional wave equations: $\ddot{x}^\mu - x''^\mu = 0$. This is not the whole story, however, because we must also take account of the constraints $T_{\alpha\beta} = 0$. Evaluated in the conformally flat gauge, these constraints are

$$\begin{aligned} T_{01} &= T_{10} = \dot{x} \cdot x' = 0 \\ T_{00} &= T_{11} = \frac{1}{2}(\dot{x}^2 + x'^2) = 0. \end{aligned} \tag{2.14}$$

Adding and subtracting gives $(\dot{x} \pm x')^2 = 0$.

2.2 Boundary Conditions

To go further, one needs to choose boundary conditions. For a closed string, which is topologically a circle, one should impose periodicity in the spatial parameter σ. Choosing its range to be π (as is conventional) $x^\mu(\sigma, \tau) = x^\mu(\sigma + \pi, \tau)$.

For an open string, which is topologically a line interval, each end can be required to satisfy either Neumann or Dirichlet boundary conditions (for each value of μ).

$$\text{Neumann}: \quad \frac{\partial x^\mu}{\partial \sigma} = 0 \quad \text{at } \sigma = 0 \text{ or } \pi \tag{2.15}$$

$$\text{Dirichlet}: \quad \frac{\partial x^\mu}{\partial \tau} = 0 \quad \text{at } \sigma = 0 \text{ or } \pi. \tag{2.16}$$

The Dirichlet condition can be integrated, and then it specifies a spacetime location on which the string ends. The only way this makes sense is if the open string ends on a physical object, which is called a D-brane. (D stands for Dirichlet.) If all the open-string boundary conditions are Neumann, then the ends of the string can be anywhere in the spacetime. The modern interpretation is that this means that spacetime-filling D-branes are present.

Let us now consider the closed-string case in more detail. The general solution of the 2d wave equation is given by a sum of "right-movers" and "left-movers":

$$x^\mu(\sigma, \tau) = x^\mu_R(\tau - \sigma) + x^\mu_L(\tau + \sigma). \tag{2.17}$$

The requirement that $x^\mu(\sigma,\tau)$ is real and periodic can be solved explicitly in terms of Fourier series:

$$x_R^\mu = \frac{1}{2}x^\mu + \ell_s^2 p^\mu(\tau-\sigma) + \frac{i}{\sqrt{2}}\ell_s \sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)} \tag{2.18}$$

$$x_L^\mu = \frac{1}{2}x^\mu + \ell_s^2 p^\mu(\tau+\sigma) + \frac{i}{\sqrt{2}}\ell_s \sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)},$$

where the expansion parameters α_n^μ, $\tilde{\alpha}_n^\mu$ satisfy $\alpha_{-n}^\mu = (\alpha_n^\mu)^\dagger$, and $\tilde{\alpha}_{-n}^\mu = (\tilde{\alpha}_n^\mu)^\dagger$, and the center-of-mass coordinate x^μ and momentum p^μ are real. The fundamental string length scale ℓ_s is related to the tension T by

$$T = \frac{1}{2\pi\alpha'}, \quad \alpha' = \ell_s^2. \tag{2.19}$$

The parameter α' is called the Regge slope.

2.3 Quantization

The analysis of closed-string left-moving modes, closed-string right-moving modes, and open-string modes are all very similar. Therefore, to avoid repetition, we will focus on the closed-string right-movers. Starting with the gauge-fixed action in eq. (2.13), the canonical momentum of the string is

$$p^\mu(\sigma,\tau) = \frac{\delta S}{\delta \dot{x}^\mu} = T\dot{x}^\mu. \tag{2.20}$$

Canonical quantization gives (setting $\hbar = 1$)

$$[p^\mu(\sigma,\tau), x^\nu(\sigma',\tau)] = -i\eta^{\mu\nu}\delta(\sigma-\sigma'). \tag{2.21}$$

This is the string generalization of $[p^\mu, x^\nu] = -i\eta^{\mu\nu}$, which encodes the Heisenberg uncertainty principle. In the terms of the Fourier components, this implies that

$$[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu} \quad \text{and} \quad [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}. \tag{2.22}$$

Recall that a quantum-mechanical harmonic oscillator can be described in terms of raising and lowering operators, usually called $a^\dagger$ and a, which satisfy $[a, a^\dagger] = 1$. We see that, aside from a normalization factor, the expansion coefficients α_{-m}^μ and α_m^μ are raising and lowering operators. There is just one problem. Because $\eta^{00} = -1$, the time components are proportional to oscillators with the wrong sign ($[a, a^\dagger] = -1$). This is potentially very bad, because such oscillators create states of negative norm, which could lead to an inconsistent quantum theory (with negative probabilities, etc.). Fortunately, in 26 dimensions, the Virasoro constraints $T_{\alpha\beta} = 0$ eliminate the negative-norm states from the physical spectrum.

The classical constraint for the right-moving closed-string modes, $(x_R')^2 = 0$, has Fourier components

$$L_m = \frac{T}{2}\int_0^\pi e^{-2im\sigma}(x_R')^2 d\sigma = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha_{m-n}\cdot\alpha_n, \tag{2.23}$$

which are called Virasoro operators. Since α_m^μ does not commute with α_{-m}^μ, L_0 needs to be normal-

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n. \tag{2.24}$$

Here $\alpha_0^\mu = \ell_s p^\mu/\sqrt{2}$, where p^μ is the momentum.

2.4 The Free String Spectrum

Recall that the Hilbert space of a harmonic oscillator is spanned by states $|n\rangle, n = 0, 1, 2, \ldots$, where the ground state, $|0\rangle$, is annihilated by the lowering operator, $a|0\rangle = 0$, and

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle. \tag{2.25}$$

Then, for a normalized ground-state ($\langle 0|0\rangle = 1$), one can use $[a, a^\dagger] = 1$ repeatedly to prove that $\langle m|n\rangle = \delta_{m,n}$ and $a^\dagger a|n\rangle = n|n\rangle$. The string spectrum (of right-movers) is given by the product of an infinite number of harmonic-oscillator Fock spaces, one for each α_n^μ, subject to the Virasoro constraints

$$\begin{aligned}(L_0 - q)|\phi\rangle &= 0 \\ L_n|\phi\rangle &= 0, \quad n > 0.\end{aligned} \tag{2.26}$$

Here $|\phi\rangle$ denotes a physical state, and q is a constant to be determined. It accounts for the arbitrariness introduced by the normal-ordering prescription used to define L_0. The L_0 equation is a generalization of the Klein–Gordon equation. It contains $p^2 = -\partial \cdot \partial$ plus oscillator terms whose eigenvalue determines the mass of the state.

It is interesting to work out the algebra of the Virasoro operators L_m, which follows from the oscillator algebra. The result, called the Virasoro algebra, is

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}. \tag{2.27}$$

The second term on the right-hand side is called the "conformal anomaly term" and the constant c is called the "central charge." In the case at hand, $c = d = 26$.

There are more sophisticated ways to describe the string spectrum (in terms of BRST cohomology), but they are equivalent to the more elementary approach presented here. In the BRST approach, gauge-fixing to the conformal gauge in the quantum theory requires the addition of world-sheet Faddeev-Popov ghosts, which turn out to contribute $c = -26$. Thus the total anomaly of the x^μ and the ghosts cancels for the choice $d = 26$. It is also necessary to set the parameter $q = 1$, so that mass-shell condition becomes $(L_0 - 1)|\phi\rangle = 0$.

Since the mathematics of the open-string spectrum is the same as that of closed-string right movers, let us now use the equations we have obtained to study the open string spectrum. (Here we are assuming that the open-string boundary conditions are all Neumann, corresponding to spacetime-filling D-branes.) The mass-shell condition is

$$M^2 = -p^2 = -\frac{1}{2\ell_s^2}\alpha_0^2 = \frac{1}{\ell_s^2}(N-1), \tag{2.28}$$

where

$$N = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n = \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n. \tag{2.29}$$

The $a^\dagger$'s and a's are properly normalized raising and lowering operators. Since each $a^\dagger a$ has eigenvalues $0, 1, 2, \ldots$, the possible values of N are also $0, 1, 2, \ldots$. The unique way to realize $N = 0$ is for all the oscillators to be in the ground state, which we denote simply by $|0; p^\mu\rangle$, where p^μ is the momentum of the state. This state has $M^2 = -1$, which is a tachyon (p^μ is spacelike). Such a faster-than-light particle is certainly not possible in a consistent quantum theory, because the vacuum would be unstable. However, for free strings (or even in perturbation theory) this instability is not visible. Since the 26-dimensional bosonic string theory is only supposed to be a warm-up exercise before considering tachyon-free superstring theories, let us proceed without worrying about tachyons.

The first excited state, with $N = 1$, corresponds to $M^2 = 0$. The only way to achieve $N = 1$ is to excite the first oscillator once:

$$|\phi\rangle = \zeta_\mu \alpha_{-1}^\mu |0; p\rangle. \tag{2.30}$$

Here ζ_μ denotes the polarization vector of a massless spin-one particle. The Virasoro constraint condition $L_1|\phi\rangle = 0$ implies that ζ_μ must satisfy $p^\mu\zeta_\mu = 0$. This ensures that the spin is transversely polarized, so there are $d-2$ independent polarization states. This agrees with what one finds for a massless Maxwell or Yang–Mills field.

Let us now turn to the closed-string spectrum. A closed-string state is described as a tensor product of a left-moving state and a right-moving state, subject to the condition that the N value of the left-moving and the right-moving state is the same. The reason for this "level-matching" condition is that we have $(L_0 - 1)|\phi\rangle = (\tilde{L}_0 - 1)|\phi\rangle = 0$. The sum $(L_0 + \tilde{L}_0 - 2)|\phi\rangle$ is interpreted as the mass-shell condition, while the difference $(L_0 - \tilde{L}_0)|\phi\rangle = (N - \tilde{N})|\phi\rangle = 0$ is the level-matching condition.

Using this rule, the closed-string ground state is just $|0\rangle \otimes |0\rangle$, which represents a spin 0 tachyon with $M^2 = -2$. (The notation no longer displays the momentum p of the state.) Again, this signals an unstable vacuum, but we will not worry about it here. Much more significant is the first excited state

$$|\phi\rangle = \zeta_{\mu\nu}(\alpha_{-1}^{\mu}|0\rangle \otimes \tilde{\alpha}_{-1}^{\nu}|0\rangle), \tag{2.31}$$

which has $M^2 = 0$. The Virasoro constraints $L_1|\phi\rangle = \tilde{L}_1|\phi\rangle = 0$ imply that $p^\mu\zeta_{\mu\nu} = p^\nu\zeta_{\mu\nu} = 0$. Such a polarization tensor encodes three distinct spin states, each of which plays a fundamental role in string theory. The symmetric part of $\zeta_{\mu\nu}$ encodes a spacetime metric field $g_{\mu\nu}$ (massless spin two) and a scalar dilaton field ϕ (massless spin zero). The $g_{\mu\nu}$ field is the graviton field, and its presence (with the correct gauge invariances) accounts for the fact that the theory contains general relativity, which is a good approximation for $E \ll 1/\ell_s$. Its vacuum value determines the spacetime geometry. Similarly, the value of ϕ determines the string coupling constant. With the usual conventions, it is given by $g_s = \langle e^\phi \rangle$.

The polarization tensor $\zeta_{\mu\nu}$ also has an antisymmetric part, which corresponds to a massless antisymmetric tensor gauge field $B_{\mu\nu} = -B_{\nu\mu}$. This field has a gauge transformation of the form

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu, \tag{2.32}$$

which is analogous to the gauge transformation rule for the Maxwell field: $\delta A_\mu = \partial_\mu \Lambda$. The gauge-invariant field strength, analogous to $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}. \tag{2.33}$$

The importance of the $B_{\mu\nu}$ field resides in the fact that the fundamental string is a source for $B_{\mu\nu}$, just as a charged particle is a source for the vector potential A_μ. Mathematically, this is expressed by a coupling to the string world sheet $\int B_{\mu\nu} dx^\mu \wedge dx^\nu$, which generalizes the coupling of a Maxwell field to the world line of a charged point particle $\int A_\mu dx^\mu$.

The number of physical states grows rapidly as a function of mass. This can be analyzed quantitatively. For the open string, let us denote the number of physical states with $\alpha' M^2 = n - 1$ by d_n. These numbers are encoded in the generating function

$$G(w) = \sum_{n=0}^{\infty} d_n w^n = \prod_{m=1}^{\infty} (1 - w^m)^{-24}. \tag{2.34}$$

The exponent 24 reflects the fact that in 26 dimensions, once the Virasoro conditions are taken into account, the spectrum is exactly what one would get from 24 transversely polarized oscillators. It is easy to deduce from this generating function the asymptotic number of states for large n, as a function of n

$$d_n \sim n^{-27/4} e^{4\pi\sqrt{n}}. \tag{2.35}$$

This asymptotic degeneracy implies that the finite-temperature partition function

$$\mathrm{tr}\,(e^{-\beta H}) = \sum_{n=0}^{\infty} d_n e^{-\beta E_n} \tag{2.36}$$

diverges for $\beta^{-1} = T > T_H$, where T_H is the Hagedorn temperature

$$T_H = \frac{1}{4\pi \ell_s}. \tag{2.37}$$

Depending on various technicalities, T_H might be the maximum possible temperature or else a critical temperature at which there is a phase transition.

2.5 Perturbation Theory

Perturbation theory is useful in a quantum theory that has a small dimensionless coupling constant, such as quantum electrodynamics (QED), since it allows one to compute physical quantities as power series expansions in the small parameter. In QED the small parameter is the fine-structure constant $\alpha \sim 1/137$. Since this is quite small, perturbation theory works very well for QED. For a physical quantity $T(\alpha)$, one computes (using Feynman diagrams)

$$T(\alpha) = T_0 + \alpha T_1 + \alpha^2 T_2 + \ldots. \tag{2.38}$$

It is the case generically in quantum field theory that expansions of this type are divergent. More specifically, they are asymptotic expansions with zero radius convergence. Nonetheless, they can be numerically useful if the expansion parameter is small. Typically there are various non-perturbative contributions (such as instantons) that have the structure

$$T_{\rm NP} \sim e^{-({\rm const.}/\alpha)}. \tag{2.39}$$

In a theory such as QCD, the couplings are small and perturbation theory is useful for large momentum processes, but for low-momentum (or "soft") processes, the coupling is large and perturbation theory is not useful. For problems of the latter type, such as computing the hadron spectrum, nonperturbative methods of computation, such as lattice gauge theory, are required.

In the case of string theory there is no particular reason why the coupling constant g_s should be small. So it is unlikely that a realistic vacuum could be analyzed accurately using perturbation theory. Until 1995 it was only understood how to analyze string theories in terms of perturbation expansions. As we will discuss later, understanding nonperturbative phenomena turned out to be very enlightening.

3 Supersymmetry and Superstrings

3.1 The RNS Model and World-Sheet Supersymmetry

Among the deficiencies of the bosonic string theory is the fact that there are no fermions. As we will see, the addition of fermions leads quite naturally to supersymmetry and hence superstrings. There are two alternative formalisms that are used to study superstrings. The original one, which grew out of the 1971 papers by Ramond [6] and by Neveu and me [7], is called the RNS formalism. In this approach, the supersymmetry of the two-dimensional world-sheet theory plays a central role.

In the RNS formalism, the world-sheet theory is based on the d functions $x^\mu(\sigma,\tau)$ that describe the embedding of the world sheet in the spacetime, just as before. However, in order to supersymmetrize the world-sheet theory, we also introduce d fermionic partner fields $\psi^\mu(\sigma,\tau)$. Note that x^μ transforms as a vector from the spacetime viewpoint, but as d scalar fields from the two-dimensional world-sheet viewpoint. The ψ^μ also transform as a spacetime vector, but as world-sheet spinors. Altogether, x^μ and ψ^μ described d supersymmetry multiplets, one for each value of μ.

The reparametrization invariant world-sheet action discussed earlier can be generalized to have local supersymmetry on the world sheet, as well. (The details will not be described here.) When one chooses the conformal gauge, $h_{\alpha\beta} = e^\phi \eta_{\alpha\beta}$, together with an appropriate fermionic gauge condition, one ends up with a world-sheet theory that has global supersymmetry supplemented by constraints. The constraints form a super-Virasoro algebra. This means that in addition to the Virasoro constraints of the bosonic string theory, there are fermionic constraints, as well.

The globally supersymmetric world-sheet action that arises in the conformal gauge takes the form [8]

$$S = -\frac{T}{2}\int d^2\sigma(\partial_\alpha x^\mu \partial^\alpha x_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu). \tag{3.1}$$

The first term is exactly the same as in eq. (2.13) of the bosonic string theory. Recall that it has the structure of d free scalar fields. The second term that has now been added is just d free massless spinor fields, with Dirac-type actions. The notation is that ρ^α are two 2×2 Dirac matrices and $\psi = \binom{\psi_-}{\psi_+}$ is a two-component Majorana spinor. The Majorana condition simply means that ψ_+ and ψ_- are real in a suitable representation of Dirac algebra. In fact, a convenient choice is one for which

$$\bar{\psi}\rho^\alpha \partial_\alpha \psi = \psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+, \tag{3.2}$$

where $\partial_\pm$ represent derivatives with respect to $\sigma^\pm = \tau \pm \sigma$. In this basis, the equations of motion are simply

$$\partial_+ \psi_-^\mu = \partial_- \psi_+^\mu = 0. \tag{3.3}$$

Thus ψ_-^μ describes right-movers and ψ_+^μ describes left-movers.

Concentrating on the right-movers ψ_-^μ, an infinitesimal global supersymmetry transformation, which is a symmetry of the gauge-fixed action, is given by

$$\begin{aligned} \delta x^\mu &= i\epsilon\psi_-^\mu \\ \delta\psi_-^\mu &= -2\partial_- x^\mu \epsilon, \end{aligned} \tag{3.4}$$

where ϵ is an infinitesimal Majorana spinor. (There is an analogous symmetry for the left-movers.) Continuing to focus on the right-movers, the Virasoro constraint is

$$(\partial_- x)^2 + \frac{i}{2}\psi_-^\mu \partial_- \psi_{\mu -} = 0. \tag{3.5}$$

The first term is what we found in the bosonic string theory, and the second term is an additional fermionic contribution. There is also an associated fermionic constraint

$$\psi_-^\mu \partial_- x_\mu = 0. \tag{3.6}$$

The Fourier modes of these constraints satisfy the super-Virasoro algebra. There is a second identical super-Virasoro algebra for the left-movers.

As in the bosonic string theory, the Virasoro algebra has conformal anomaly terms proportional to a central charge c. As in that theory, each component of x^μ contributes $+1$ to the central charge, for a total of d, while (in the BRST quantization approach) the reparametrization symmetry ghosts contribute -26. But now there are additional contributions. Each component of ψ^μ gives $+1/2$, for a total of $d/2$, and the local supersymmetry ghosts contribute $+11$. Adding all of this up, gives a grand total of $c = \frac{3d}{2} - 15$. Thus, we see that the conformal anomaly cancels for the specific choice $d = 10$. This is the preferred critical dimension for superstrings, just as $d = 26$ is the critical dimension for bosonic strings. For other values the theory has a variety of inconsistencies.

Let us now consider boundary conditions for $\psi^\mu(\sigma, \tau)$. (The story for x^μ is exactly as before.) First, let us consider open-string boundary conditions. For the action to be well-defined, it turns out that one must set $\psi_+ = \pm\psi_-$ at the two ends $\sigma = 0, \pi$. An overall sign is a matter of convention, so we can set $\psi_+^\mu(0,\tau) = \psi_-^\mu(0,\tau)$, without loss of generality. But this still leaves two possibilities for the other end, which are called R and NS:

$$\begin{aligned} \text{R} &: \ \psi_+^\mu(\pi,\tau) = \psi_-^\mu(\pi,\tau) \\ \text{NS} &: \ \psi_+^\mu(\pi,\tau) = -\psi_-^\mu(\pi,\tau). \end{aligned} \tag{3.7}$$

Combining these with the equations of motion $\partial_- \psi_+ = \partial_+ \psi_- = 0$, allows us to express the general solutions as Fourier series

$$\begin{aligned} \text{R}: \quad \psi^\mu_\pm &= \frac{1}{\sqrt{2}} \sum_{n \in \mathbf{Z}} d^\mu_n e^{-in(\tau \pm \sigma)} \\ \text{NS}: \quad \psi^\mu_\pm &= \frac{1}{\sqrt{2}} \sum_{r \in \mathbf{Z}+1/2} b^\mu_r e^{-ir(\tau \pm \sigma)}. \end{aligned} \tag{3.8}$$

The Majorana condition implies that $d^\mu_{-n} = d^{\mu\dagger}_n$ and $b^\mu_{-r} = b^{\mu\dagger}_r$. Note that the index n takes integer values, whereas the index r takes half-integer values $(\pm\frac{1}{2}, \pm\frac{3}{2}, \ldots)$. In particular, only the R boundary condition gives a zero mode.

Canonical quantization of the free fermi fields $\psi^\mu(\sigma, \tau)$ is very standard and straightforward. The result can be expressed as anticommutation relations for the coefficients d^μ_m and b^μ_r:

$$\begin{aligned} \text{R} \quad &: \quad \{d^\mu_n, d^\nu_n\} = \eta^{\mu\nu} \delta_{m+n,0} \qquad m, n \in \mathbf{Z} \\ \text{NS} \quad &: \quad \{d^\mu_r, d^\nu_s\} = \eta^{\mu\nu} \delta_{r+s,0} \qquad r, s \in \mathbf{Z} + \frac{1}{2}. \end{aligned} \tag{3.9}$$

Thus, in addition to harmonic oscillator operators α^μ_m that appear as coefficients in
llator operators d^μ_m or b^μ_r that appear as coefficients in
ure $\{b, b^\dagger\} = 1$ is very simple. It describes a two-state
The b's or d's with negative indices can be regarded as
indices as lowering operators, just as we did for the α^μ_n.
$; p\rangle$ satisfies

$$= b^\mu_r |0; p\rangle = 0, \quad m, r > 0 \tag{3.10}$$

of how we defined the ground state in the bosonic string
acting with the α and b raising operators are spacetime
state, defined as we have done here, is again a tachyon.
by which this tachyon can (and must) be removed from

that satisfy the algebra

$$\{d^\mu_0, d^\nu_0\} = \eta^{\mu\nu}. \tag{3.11}$$

ac algebra. Thus the d_0's should be regarded as Dirac
$\overline{2})$ and all states in the R sector should be spinors in order to furnish representation spaces on which these operators can act. The conclusion, therefore, is that whereas all string states in the NS sector are spacetime bosons, all string states in the R sector are spacetime fermions.

The zero mode of the fermionic constraint $\psi^\mu \partial_- x_\mu = 0$ gives a wave equation for (fermionic) strings in the Ramond sector, $F_0|\psi\rangle = 0$, which is called the Dirac–Ramond equation. In terms of the oscillators

$$F_0 = \alpha_0 \cdot d_0 + \sum_{n \neq 0} \alpha_{-n} \cdot d_n. \tag{3.12}$$

The zero-mode piece of F_0, $\alpha_0 \cdot d_0$, has been isolated, because it is just the usual Dirac operator, $\gamma^\mu \partial_\mu$, up to normalization. (Recall that $\alpha_{0\mu}$ is proportional to $p_\mu = -i\partial_\mu$, and d^μ_0 is proportional to the Dirac matrices γ^μ.) The fermionic ground state $|\psi_0\rangle$, which satisfies

$$\alpha^\mu_n |\psi_0\rangle = d^\mu_n |\psi_0\rangle = 0, \quad n > 0, \tag{3.13}$$

satisfies the wave equation

$$\alpha_0 \cdot d_0 |\psi_0\rangle = 0, \tag{3.14}$$

which is precisely the massless Dirac equation. Hence the fermionic ground state is a massless spinor.

In the closed-string case, the physical states are obtained by tensoring right-movers and left-movers, each of which are mathematically very similar to the open-string spectrum. This means that there are four distinct sectors of closed-string states: NS⊗NS and R⊗R describe spacetime bosons, whereas NS⊗R and R⊗NS describe spacetime fermions.

3.2 Gravity and Unification

The original string theory described above requires 26 dimensions (25 spatial dimensions and one time dimension). Also, the particle spectrum includes tachyons, and it does not contain any fermions, one of the two important classes of particles in quantum theories. For these reasons, it was clear that this string theory could not be a realistic theory of the strong interactions. The RNS theory contains fermions, which is progress, but it requires ten dimensions, so clearly it is not the correct theory of the strong interactions either. Another problem is the occurrence of massless particles, which conflicts with the fact that all particles that have strong nuclear interactions are massive.

Among the massless closed-string states (of either theory) there is one that has spin two. In 1974, it was shown by Scherk and me [9], and independently by Yoneya [10], that this particle interacts like a graviton, so the theory actually includes general relativity. This led us to propose that string theory should be used for gravity and unification rather than for hadrons. This implied, in particular, that the string length scale should be comparable to the Planck length, rather than the size of hadrons (10^{-13} cm) as we had previously assumed.

This proposal had two immediate benefits: 1) Quantum contributions in field theory descriptions of gravity have high-energy (or ultraviolet) divergences and therefore give infinite values for physical quantities. On the other hand, string theory is free from ultraviolet divergences. 2) In the context of the original goal of string theory – to explain hadron physics – extra dimensions are unacceptable. However, in a theory that incorporates general relativity, the geometry of spacetime is determined dynamically. Thus one could imagine that the theory admits consistent quantum solutions in which the six extra spatial dimensions form a compact space, too small to have been observed. The natural first guess is that the size of this space should be comparable to the string scale and the Planck length.

3.3 Spacetime Supersymmetry

In 1976 Gliozzi, Scherk, and Olive [11] noted that the RNS spectrum admits a consistent truncation (called the GSO projection), which is necessary for the consistency of the interacting theory. In the NS sector, the GSO projection keeps states with an odd number of b-oscillator excitations, and removes states with an even number of b-oscillator excitations. Once this rule is implemented the spectrum of allowed masses is integral ($M^2 = 0, 1, 2, \ldots$). In particular, the bosonic ground state is now massless, so the spectrum no longer contains a tachyon. The GSO projection also acts on the R sector, where there is an analogous restriction that amounts to imposing a chirality projection on the spinors. The claim is that the truncated theory has spacetime supersymmetry.

If there is spacetime supersymmetry, then there should be an equal number of bosons and fermions at every mass level. Let us denote the number of bosonic states with $M^2 = n$ by $d_{\rm NS}(n)$ and the number of fermionic states with $M^2 = n$ by $d_{\rm R}(n)$. Then we can encode these numbers in generating functions

$$f_{\rm NS}(w) = \sum_{n=0}^{\infty} d_{\rm NS}(n) w^n = \frac{1}{2\sqrt{w}} \left(\prod_{m=1}^{\infty} \left(\frac{1 + w^{m-1/2}}{1 - w^m} \right)^8 - \prod_{m=1}^{\infty} \left(\frac{1 - w^{m-1/2}}{1 - w^m} \right)^8 \right) \tag{3.15}$$

and

$$f_{\rm R}(w) = \sum_{n=0}^{\infty} d_{\rm R}(n) w^n = 8 \prod_{m=1}^{\infty} \left(\frac{1 + w^m}{1 - w^m} \right)^8 . \tag{3.16}$$

The 8's in the exponents refer to the number of transverse directions in ten dimensions. The effect of the GSO projection is the subtraction of the second term in $f_{\rm NS}$ and reduction of coefficient in

$f_{\rm R}$ from 16 to 8. In 1829, Jacobi discovered the formula[2]

$$f_{\rm R}(w) = f_{\rm NS}(w). \tag{3.17}$$

For him this relation was an obscure curiosity, but we now see that it provides strong evidence for supersymmetry of the GSO-projected string theory in ten dimensions.

A complete proof of spacetime supersymmetry for the interacting string theory was constructed by Green and me five years after the GSO paper [12]. We developed an alternative world-sheet theory to describe the GSO-projected theory based on world-sheet fields X^μ and θ^a, representing ten-dimensional superspace, in which supersymmetry was manifest. In this formulation the action describes the embedding of the world-sheet in superspace.

Spacetime supersymmetry is the major prediction of superstring theory that could be confirmed experimentally at accessible energies, that has not been discovered already. A variety of arguments, not specific to string theory, suggest that the characteristic energy scale associated to supersymmetry breaking should be related to the electroweak scale, in other words in the range 100 GeV – 1 TeV. The symmetry implies that each of the known elementary particles should have a supersymmetry partner particle, whose mass is in this general range. This means that some of these superpartners should be observable at the CERN Large Hadron Collider (LHC), which is currently under construction and is scheduled to begin operating in 2007. This machine collides two 7 TeV beams of protons. There is a slight chance that the Fermilab Tevatron, which is currently in operation with collisions of 1 TeV beams, could discover superparticles first.

In most versions of phenomenological supersymmetry there is a multiplicatively conserved quantum number called R-parity. All known particles have even R-parity, whereas their superpartners have odd R-parity. Conservation of R-parity implies that the superparticles must be pair-produced in particle collisions. It also implies that the lightest supersymmetry particle (or LSP) should be absolutely stable. It is not known with certainty which particle is the LSP, but one popular guess is that it is a "neutralino." This is an electrically neutral fermion that is a quantum-mechanical mixture of the partners of the photon, Z^0, and neutral Higgs particles. Such an LSP would interact very weakly, like a neutrino. A neutralino LSP is of considerable interest, since it is an excellent dark matter candidate.[3] Searches for a class of possible dark matter particles called WIMPS (weakly interacting massive particles) could discover a neutralino LSP some day, though current experiments might not have sufficient detector volume to compensate for the exceedingly small collision cross sections.

3.4 Superstrings

The first superstring revolution began in 1984 with the discovery that quantum mechanical consistency of a ten-dimensional theory with $\mathcal{N} = 1$ supersymmetry requires a local Yang–Mills gauge symmetry based on one of two possible Lie groups: $SO(32)$ or $E_8 \times E_8$ [13]. Only for these two choices do certain quantum mechanical *anomalies* cancel. The fact that these groups were singled out caused a lot of excitement, because in ordinary quantum field theory there is no mathematical principle that makes one group better than any other. The fact that only these groups are possible suggested that string theory has a very constrained structure, and therefore it might be very predictive.

When one uses the supersymmetric string formalism for both left-moving modes and right-moving modes, the supersymmetries associated with the left-movers and the right-movers can have either opposite handedness or the same handedness. These two possibilities give different theories called the type IIA and type IIB superstring theories, respectively. A third possibility, called type I superstring theory, can be derived from the type IIB theory by modding out by its left-right symmetry (a procedure called orientifold projection). The strings that survive this projection are unoriented.

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[2] He used a different notation, of course.

[3] Most of the mass of the universe consists of nonluminous matter, which has not yet been directly observed. Its existence is inferred from its gravitational effects.

A more surprising possibility is to use the formalism of the 26-dimensional bosonic string for the left-movers and the formalism of the 10-dimensional supersymmetric string for the right-movers. The string theories constructed in this way are called "heterotic" [14]. The mismatch in spacetime dimensions may sound strange, but it is actually okay. The extra 16 left-moving dimensions must describe a torus with very special properties to give a consistent theory. There are precisely two distinct tori that have the required properties, and they correspond to the Lie groups $SO(32)$ or $E_8 \times E_8$.

Altogether, there are five distinct superstring theories, each in ten dimensions. Three of them, the type I theory and the two heterotic theories, have $\mathcal{N} = 1$ supersymmetry in the ten-dimensional sense. The minimal spinor in ten dimensions has 16 real components, so these theories have 16 conserved supercharges. The type I superstring theory has the gauge group $SO(32)$, whereas the heterotic theories realize both $SO(32)$ (the HO theory) and $E_8 \times E_8$ (the HE theory). The other two theories, type IIA and type IIB, have $\mathcal{N} = 2$ supersymmetry (32 supercharges).

In each of these five superstring theories there are consistent perturbation expansions of physical quantities. In four of the five cases (heterotic and type II) the fundamental strings are oriented and unbreakable. As a result, these theories have particularly simple perturbation expansions. Specifically, there is a unique Feynman diagram at each order of the expansion. The Feynman diagrams depict string world sheets, and therefore they are two-dimensional surfaces. For these four theories the unique L-loop diagram is a genus-L Riemann surface, which can be visualized as a sphere with L handles. External (incoming or outgoing) particles are represented by N points (or "punctures") on the Riemann surface. A given diagram represents a well-defined integral of dimension $6L + 2N - 6$. This integral has no ultraviolet divergences. Type I superstrings are unoriented and breakable. As a result, the perturbation expansion is more complicated for this theory, and there are various world-sheet Feynman diagrams at each order. The separate diagrams have divergences that cancel when they are combined correctly.

3.5 Compactification of Extra Dimensions

All five superstring theories require that spacetime should have ten dimensions, which is six more than are observed. The reason this is not a fatal problem is that these theories contain general relativity, and therefore the geometry of spacetime is determined dynamically. In other words, the spacetime geometry must be part of a complete solution of the equations of the theory. This is a severe constraint, but it still leaves many possibilities. Among these possibilities there are ones in which the ten dimensions consist of a product of four-dimensional Minkowski spacetime with a compact six-dimensional manifold K. If K has a typical size a, then by general principles of quantum mechanics, its existence would be unobservable for energies below $E_a = \hbar c/a$. The most natural guess is that this compactification scale should be comparable to the unification scale or the string scale.

The possibilities for K are quite limited, especially if one requires that there be some supersymmetry below the compactification scale. A class of manifolds K that has been studied a great deal are called Calabi–Yau spaces [15].[4] They have properties that ensure that $1/4$ of the original supersymmetry is unbroken at low energies. In particular, starting with the HE theory compactified on a suitably chosen Calabi–Yau space one can come quite close to making contact with a realistic supersymmetric grand-unified theory. In the late 1980s such scenarios received a great deal of attention. More recently, it has been recognized that there are a variety of other ways that superstrings could give rise to a realistic model. Some of them are based on type II superstrings.

4 Recent Developments in Superstring Theory

The discovery that superstring theory can consistently unify gravity with the other forces in a quantum framework was an important development. However, the realization that there are five different superstring theories was somewhat puzzling. Certainly, there is only one physical universe

[4] A Calabi–Yau space is a special type of six-dimensional space, which can described using three complex coordinates. More precisely, it is a Kähler manifold of $SU(3)$ holonomy.

that we can ever hope to observe, so it would be most satisfying if there were only one possible theory. In the late 1980s it was realized that when extra dimensions are compact there is a property known as T duality that relates the two type II theories and the two heterotic theories, so that they shouldn't really be regarded as distinct theories. T duality can be understood within the framework of perturbation theory.

Further progress required understanding nonperturbative phenomena, something that was achieved in the 1990s. Nonperturbative S dualities and the opening up of an eleventh dimension led to new identifications. Once all of these correspondences are taken into account, one ends up with the best possible conclusion. There really is a unique underlying theory, which has no arbitrary adjustable dimensionless parameters.

4.1 T Duality

String theory exhibits many strange and surprising properties. One that was discovered in the late 1980s is called T duality.[5] In many cases, T duality implies that two different geometries for the extra dimensions, K and $\tilde{K}$, are physically equivalent! In the simplest example, a circle of radius R is equivalent to a circle of radius ℓ_s^2/R, where (as before) ℓ_s is the fundamental string length scale.

Let us sketch an argument that should make this duality plausible. When there is a circular extra dimension, the momentum along that direction is quantized: $p = n/R$, where n is an integer. Using the relativistic energy formula $E^2 = M^2 + \sum_i (p_i)^2$, one sees that the momentum along the circular dimension can be interpreted as contributing an amount $(n/R)^2$ to the mass squared as measured by an observer in the noncompact dimensions. This is true whether one is considering point particles, strings, or any other kinds of objects. Particle states with $n \neq 0$ are usually referred to as *Kaluza–Klein excitations.*

In the special case of closed strings, there is a second kind of excitation that can also contribute to the mass squared. Namely, the string can be wound around the circle, so that it is caught up on the topology of the space. The contribution to the mass squared is the square of the tension $T = (2\pi\ell_s^2)^{-1}$ times the length of wrapped string, which is $2\pi Rm$, if it wraps m times. Multiplying, the contribution to the mass squared is $(Rm/\ell_s^2)^2$. These are referred to as *winding mode excitations.*

Now we can make the key observation: Under T duality the role of Kaluza–Klein excitations and winding-mode excitations are interchanged. Note that the two contributions to the mass squared are exchanged if one interchanges m and n and at the same time sends $R \to \ell_s^2/R$.

T duality typically relates two different theories. Two particularly important examples are

$$\text{IIA} \leftrightarrow \text{IIB} \quad \text{and} \quad \text{HE} \leftrightarrow \text{HO}.$$

Therefore type IIA and type IIB (also HE and HO) should be regarded as a single theory. More precisely, they represent opposite ends of a continuum of geometries as one varies the radius of a circular dimension. This radius is not a parameter of the underlying theory. Rather, it arises as the value of a scalar field, and therefore it is determined dynamically.

There are also fancier examples of T-duality equivalences. For example, there is an equivalence of type IIA superstring theory compactified on a Calabi–Yau space and type IIB compactified on the "mirror" Calabi-Yau space. This mirror pairing of topologically distinct Calabi-Yau spaces is a striking discovery made by physicists that has subsequently been explored by mathematicians.

T duality might play a role in cosmology, since it suggests a possible way for a big crunch to turn into a big bang. The heuristic idea is that a contracting space when it becomes smaller than the string scale can be reinterpreted as an expanding space that is larger than the string scale, without the need for any exotic forces to halt the contraction. Unfortunately, we are not yet able to analyze such time-dependent scenarios reliably.

[5] The letter T has no particular significance. It was the symbol used by some authors for one of the low energy fields.

4.2 S Duality

Another kind of duality – called S duality – was discovered as part of the "second superstring revolution" in the mid 1990s [16], [17]. (There had been some related proposals earlier [18], [19], [20].) S duality relates the string coupling constant g_s to $1/g_s$ in the same way that T duality relates R to $1/R$. The two basic examples are

$$\text{I} \leftrightarrow \text{HO} \quad \text{and} \quad \text{IIB} \leftrightarrow \text{IIB}.$$

Thus, given our knowledge of the small g_s behavior of these theories, we learn how these three theories behave when $g_s \gg 1$. For example, strongly coupled type I theory is equivalent to the weakly coupled SO(32) heterotic theory. In the type IIB case the theory is related to itself, so one is actually dealing with a symmetry. Let us examine this case in a little more detail.

Type IIB superstring theory contains two massless scalar fields, the dilaton ϕ and the axion χ, which are conveniently combined in a complex field

$$\rho = \chi + ie^{-\phi}. \tag{4.1}$$

The low-energy (or supergravity) approximation has an $SL(2,R)$ symmetry that transforms this field nonlinearly:

$$\rho \to \frac{a\rho + b}{c\rho + d}, \tag{4.2}$$

where a, b, c, d are real numbers satisfying $ad - bc = 1$. However, in the exact theory this symmetry is broken to the discrete subgroup $SL(2,Z)$ [16], which means that a, b, c, d are restricted to be integers. Defining the vacuum value of the ρ field to be

$$\langle\rho\rangle = \frac{\theta}{2\pi} + \frac{i}{g_s}, \tag{4.3}$$

the $SL(2,Z)$ symmetry transformation $\rho \to \rho + 1$ implies that θ is an angular coordinate. Moreover, in the special case $\theta = 0$, the symmetry transformation $\rho \to -1/\rho$ takes $g_s \to 1/g_s$. This symmetry, which is called S duality, implies that coupling constant g_s is equivalent to coupling constant $1/g_s$, so that, in the case of type IIB superstring theory, the weak coupling expansion and the strong coupling expansion are identical!

An analogous S-duality transformation relates the type I superstring theory to the $SO(32)$ heterotic string theory. In that case there is no axion, and so the only transformation is $\phi_{\rm I} \to -\phi_{\rm H}$, which implies that g_s in one theory corresponds to $1/g_s$ in the other.

4.3 D-Branes

When studied nonperturbatively, one discovers that superstring theory contains various p-branes, objects with p spatial dimensions, in addition to the fundamental strings. Their role only became well understand when nonperturbative aspects of string theory came under control in the mid 1990s. The reason for this is that all of the p-branes, with the single exception of the fundamental string (which is a 1-brane), become infinitely heavy as $g_s \to 0$, and therefore they do not play a role in perturbation theory. On the other hand, when the coupling g_s is not small, this distinction no longer significant. When that is the case, all of the p-branes are just as important as the fundamental strings. In other words, what makes fundamental strings "fundamental" is the weak coupling expansion.

Each of the type II and heterotic theories contains a stable 5-brane (sometimes called an NS5-brane), whose tension is proportional to $1/g_s^2$. In addition, the type I and II superstring theories contain a class of p-branes called D-branes, whose tension is proportional $1/g_s$. Another characteristic feature of stable D-branes in the type I and II superstring theories is that they are sources for gauge fields in the RR sector [21].

As we mentioned earlier, the defining property of D-branes is that fundamental strings can end on them. This implies that quantum field theories of the Yang–Mills type, like the standard model, reside on D-branes [22]. The Yang-Mills fields arise as the massless modes of open strings attached to the D-branes. An interesting possibility is that the reason we experience four spacetime dimensions is because we are confined to live on three-dimensional D-branes (D3-branes), which are embedded in a spacetime with six additional spatial directions. In this type of a picture, the six transverse dimensions do not necessarily need to be small. Model-building along these lines is one of the approaches that is being explored.

4.4 M Theory

S duality tells us how three of the five original superstring theories behave at strong coupling. This raises the question: What happens to the other two superstring theories – type IIA and HE – when g_s is large? The answer, which came as quite a surprise, is that they grow an eleventh dimension of size $g_s\ell_s$. This new dimension is a circle in the type IIA case [23], [17] and a line interval in the HE case [24]. When the eleventh dimension is large, one is outside the regime of perturbative string theory, and new techniques are required. This calls for a new type of quantum theory, for which Witten has proposed the name M theory.[6]

M theory has not been yet been formulated in the most general setting. However, quite a lot is known about it. For one thing it is approximated at low energies by eleven-dimensional supergravity, a theory that was formulated long ago [25]. A fundamental fact about 11-dimensional supergravity is that it contains a massless three-form gauge field. Because of this, it turns out that M theory contains two types of stable p-branes: a 2-brane and a 5-brane. They couple electrically and magnetically, respectively, to the three-form gauge field.

There is a conjecture for an exact quantum mechanical description of M theory, that goes by the name of Matrix theory [26]. This proposal gives a dual description of M theory in flat 11-dimensional spacetime in terms of the quantum mechanics of $N \times N$ matrices in the large N limit. When n of the spatial dimensions are compactified on a torus, the dual theory becomes a quantum field theory in n spatial dimensions (plus time). There is compelling evidence that this conjecture is correct when n is not too large. However, it is unclear how to generalize it to other geometries, so it is not the whole story.

Recall that the type IIA and type IIB superstring theories are T dual, meaning that if they are compactified on circles of radii R_A and R_B, respectively, one obtains equivalent theories for the identification $R_A R_B = \ell_s^2$. Moreover, we have just learned that the type IIA theory is actually M theory compactified on a circle, a fact that encodes nonperturbative information. It turns out to be very useful to combine these two facts and to consider the duality between M theory compactified on a torus, so that the eleven-dimensional spacetime is $R^9 \times T^2$, and type IIB superstring theory compactified on a circle, so that the ten-dimensional spacetime is $R^9 \times S^1$. These should be precisely equivalent.

A torus can be described as the complex plane modded out by the equivalence relations $z \sim z + w_1$ and $z \sim z + w_2$. Up to conformal equivalence, the periods w_1 and w_2 can be replaced by 1 and τ, with Im $\tau > 0$. In this characterization τ and $\tau' = (a\tau + b)/(c\tau + d)$, where a, b, c, d are integers satisfying $ad - bc = 1$, describe equivalent tori. Thus a torus is characterized by a modular parameter τ and an $SL(2, Z)$ modular group. The natural, and correct, conjecture at this point is that one should identify the modular parameter τ of the M theory torus with the parameter ρ that characterizes the type IIB vacuum [27, 28]. Then the duality of M theory and type IIB superstring theory gives a geometrical explanation of the nonperturbative S duality symmetry of the IIB theory: the transformation $\rho \to -1/\rho$, which sends $g_s \to 1/g_s$ in the IIB theory, corresponds to interchanging the two cycles of the torus in the M theory description.

Another interesting fact about the IIB theory is that it contains an infinite family of strings labeled by a pair of integers (p, q) with no common divisor [27]. The $(1, 0)$ string can be identified as the fundamental IIB string, while the $(0, 1)$ string is the D-string. (A D-string is a D1-brane.) From this viewpoint, a (p, q) string can be regarded as a bound state of p fundamental strings and q D-strings [22]. These strings have a very simple interpretation in the dual M theory description.

[6] He suggests that M should represent “mysterious” or “magical.” Others have suggested that M could stand for “membrane” or “mother.”

They correspond to an M-theory 2-brane with one of its cycles wrapped around a (p, q) homology cycle of the torus. The minimal length of such a cycle is proportional to $|p + q\tau|$, and thus (using $\tau = \rho$) one finds that the tension of a (p, q) string is given by

$$T_{p,q} = 2\pi|p + q\rho|m_s^2. \tag{4.4}$$

One can try to construct a realistic four-dimensional theory starting from M theory. Since this means starting in eleven dimensions, it is necessary to choose a suitable 7-manifold for the extra dimensions. The way to get $\mathcal{N} = 1$ supersymmetry in four dimensions is to require that the 7-manifold have G_2 holonomy. The study of G_2 manifolds is more difficult and less well understood than that of Calabi–Yau manifolds. It is plausible that some models constructed in this way will turn out to be dual to ones constructed by Calabi–Yau compactification of the HE theory. Such relations are interesting, because the M theory picture would allow one to understand phenomena that are nonperturbative in the heterotic picture.

4.5 Black Hole Entropy

The gravitational field generated by a large number of coincident D-branes causes warpage of the spacetime geometry and creates an event horizon. This generalizes the usual story of black holes to higher dimensional objects in higher dimensional spacetimes.

One of Hawking's important discoveries was that black holes behave like thermodynamic objects with a well-defined temperature and entropy. The entropy is given (in gravitational units) by 1/4 the area of the event horizon. In quantum theory, an entropy S ordinarily means that there are a large number of quantum states (namely, $\exp S$) that are contributing. So a natural question is whether this rule also applies to black holes. D-branes provide a set-up in which this question can be investigated.

In special cases, starting with an example in five dimensions that was analyzed by Strominger and Vafa [29], one can count the quantum microstates associated with D-brane excitations and compare the result with the Bekenstein-Hawking entropy formula. Although many examples have been studied and no discrepancies have been found (aside from tiny corrections that are expected), this correspondence has not yet been been derived in full generality. The problem is that one needs to extrapolate from the weakly coupled D-brane picture to the strongly coupled black hole one, and mathematical control of this extrapolation is only straightforward when there is a generous measure of unbroken supersymmetry. This is only true for specially chosen examples. Even so, it is fair to say that studies of D-branes have led to a much deeper understanding of the thermodynamic properties of black holes in terms of string theory microphysics, a fact that is one of the most notable successes of string theory so far.

4.6 AdS/CFT Duality

In a remarkable development, Maldacena conjectured that the quantum field theory that lives on a collection of D3-branes (in the IIB theory) is actually equivalent to type IIB string theory in the geometry that the gravitational field of the D3-branes creates [30]. He also proposed several other analogous M theory dualities. These dualities are sometimes referred to as AdS/CFT dualities, where AdS stands for anti de Sitter space and CFT stands for conformal field theory.[7] The near-horizon geometry produced by the D-branes is a product of an anti de Sitter space and a sphere, and the dual field theory is conformally invariant.

In the example that arises from considering N coincident D3-branes in the type IIB theory, one obtains a duality between $SU(N)$ Yang-Mills theory with $\mathcal{N} = 4$ supersymmetry in four dimensions and the type IIB string theory in a ten-dimensional geometry given by a product of a five-dimensional Anti-de Sitter space (AdS_5) and a five-dimensional sphere (S^5). There are N units of five-form flux threading the five sphere. Also, the radius R of the sphere and the Anti-de Sitter

[7] AdS space is a maximally symmetric geometry with negative scalar curvature. A CFT is invariant under the group of conformal transformations. This is an extension of the Poincaré group that includes transformations that rescale distances.

space and the string coupling constant g_s are related to the Yang–Mills theory coupling constant g_{YM} and gauge group $SU(N)$ by the relations

$$(R/l_s)^4 = g_{YM}^2 N \tag{4.5}$$

and

$$g_s = g_{YM}^2. \tag{4.6}$$

The implications of this duality for correlation functions were spelled out in [31], [32]. This astonishing proposal has been extended and generalized in a couple thousand subsequent papers. Some of this work is reviewed in [33]. While we can't hope to convince you here that these dualities are sensible, we can point out that the first check is that the symmetries match. An important ingredient in this matching is the fact that the symmetry group of Anti de Sitter space in $n+1$ dimensions is $SO(n,2)$, which is the same as that of the conformal group in n dimensions. The general idea is that the n-dimensional CFT is associated to the boundary of the $(n+1)$-dimensional AdS space.[8] The extra "radial" direction of the AdS space corresponds to size (or scale) in the dual CFT.

This type of identification is an example of a "holographic" duality, since it is analogous to representing three-dimensional space on a two-dimensional emulsion. The possibility of such holographic correspondences arose in earlier studies of black holes in work of 't Hooft and Susskind [34], [35]. Holographic dualities can also be generalized to cases where the conformal symmetry is broken. One way of doing this is by giving masses to some of the fields in the CFT.

5 Problems and Prospects

In this final section, we discuss some of the important issues that still need to be resolved, if string theory is to achieve its lofty goals. These goals are two-fold: to develop a complete theoretical description of fundamental microphysics and to understand the origin, evolution, and fate of the universe. As will be evident, the issues discussed below represent formidable challenges, and the solution of any one of them would be an important achievement. I am optimistic that breakthroughs will be achieved in the coming years that will resolve, or at least recast, some of these questions. This optimism is based both on a belief in the intrinsic beauty of the underlying theory and on a very high regard for the many clever people who are studying it. At the same time, I think it is not at all clear whether the end of the quest will ever be achieved.

5.1 Find a Complete Formulation of the Theory

There are techniques for identifying large classes of superstring and M theory vacua, and describing them exactly, but there is not yet a succinct and compelling formulation of the underlying theory that gives rise to these vacua. Even though such a formulation is not known, it is quite clear from what we do know that it should be completely unique, with no adjustable parameters or other arbitrariness.

Many things that we usually take for granted, such as the existence of a spacetime manifold, are likely to be understood as emergent properties of specific vacua rather than identifiable features of the underlying theory. If this is correct, then the missing formulation of the theory must be quite unlike any previous theory. Usual approaches based on quantum fields depend on the existence of an ambient spacetime manifold. It is not clear what the basic degrees of freedom should be in a theory that does not assume a spacetime manifold at the outset.

5.2 Understand the Energy Density of the Vacuum

In a quantum theory that contains gravity, such as string theory, the cosmological constant, Λ, which characterizes the energy density of the vacuum, is (at least in principle) a computable quantity. This energy (sometimes called *dark energy*) has recently been measured to fairly good

[8] The boundary is actually located at infinity.

accuracy, and the result is surprising: it accounts for about 70% of the total mass/energy in the universe.[9] Moreover, the sign of the cosmological constant is such that that the self-gravitation of the vacuum is repulsive. This accounts for the observed acceleration in the expansion of the present-day Universe.

The observed value of the cosmological constant/dark energy is important for cosmology, but it is extremely tiny when expressed in Planck units (about 10^{-120}). Therefore, a static Lorentz invariant Minkowski spacetime, which has a vanishing vacuum energy, is surely an excellent approximation to the real world for particle physics purposes. We can achieve an exact cancellation between the contributions of bosons and fermions to the vacuum energy when there is unbroken supersymmetry, but no known principle ensures a near-perfect cancellation when supersymmetry is broken.

Many imaginative proposals have been made to solve this problem, but none of them has gained a wide following. In my view one should first understand how to derive $\Lambda = 0$ when supersymmetry is broken, and then later try to account for the tiny nonzero value that is actually observed. A radically different viewpoint that has gained in popularity recently is that string theory can accommodate almost any value of Λ, but only solutions for which Λ is sufficiently small can support life. So, if it were much larger, we wouldn't be here to ask the question. This type of reasoning is called "anthropic."

5.3 Explain Elementary Particle Physics

Even though the underlying theory is unique, it admits an enormous number of different solutions (or quantum vacua). Assuming that the theory is correct, one of these solutions should describe the real world. The challenge is to find it.

The Universe we inhabit is changing with time, and (as we have just discussed) it appears to have a small cosmological constant. However, for the purpose of describing elementary particle physics it is surely an excellent approximation to assume that the Universe is static with no cosmological constant. In this framework, we would like to understand what are all the possible quantum vacua of the fundamental theory, and which one gives a correct description of the microscopic world of particle physics. This is a tall order.

Many classes of consistent supersymmetric vacua, often with a large number of parameters (called moduli), have been found. Techniques for stabilizing the moduli have been developed, which is important for preventing the occurrence of unobserved massless scalar particles. The analysis becomes more difficult as the amount of supersymmetry that is broken increases. Vacua in which all of the supersymmetry is broken (as in the real world) are particularly challenging. In addition to the issue of the cosmological constant, one must also establish their quantum stability. Nonsupersymmetric solutions that are stable in the classical approximation can be destabilized by quantum corrections.

Presumably, if one had a complete list of all possible quantum vacua, one of them would be the "right one." It would be marvelous to identify this vacuum, but we would also like to understand *why* it is the right one. Is it picked out by some special mathematical property, or is it just an environmental accident of our particular corner of the Universe? The way this question plays out will be important in determining the extent to which the observed world of particle physics can be deduced from first principles.

5.4 Understand the Structure of Spacetime and the Status of Quantum Mechanics

Hawking has suggested that when matter falls into black holes and eventually comes back out as thermal radiation (called Hawking radiation), quantum coherence is lost [36]. In other words, an initially pure quantum state can evolve into a mixed state, in violation of the basic tenets of quantum mechanics. Most string theorists are convinced that this argument is not correct, but it is difficult to explain precisely how string theory evades it.

[9]It is possible that something other than a cosmological constant is responsible for the dark energy. However, even if that is the case, the data suggests that a cosmological constant is a good approximation.

Singularities in the geometry of spacetime are a common feature of nontrivial solutions to general relativity. In the case of black holes they are shielded behind a horizon. However, they can also occur unshielded by a horizon, in which case one speaks of a *naked singularity*. Not only are singularities places where general relativity breaks down, but even worse they undermine the Cauchy problem – the ability to deduce the future from initial data. Another important issue, which may be related, is to understand what ensures that the spacetime geometry has no closed timelike curves, since their presence creates causal paradoxes.

The situation in string theory is surely better. Strings respond to spacetime differently from point particles. Certain classes of spacelike singularities, which would not be acceptable in general relativity, are known to be entirely harmless in string theory. However, there are other important types of singularities that are not spacelike, for which current string theory technology is unable to say what happens. Perhaps some of them are acceptable and others are forbidden, but it remains to be explained which is which and how this works.

My impression is that a number of people are making important progress in addressing this class of issues. So I am optimistic that they will be resolved relatively soon.

5.5 Understand the Origin and Evolution of the Universe

Within the past few years people have started to carefully analyze time dependent solutions to string theory. This is important for addressing the questions raised in the preceding subsection. It is also important for cosmological applications. The first goal is to construct examples – even ones that are unrealistic – that can be analyzed in detail, and that do not lead to pathologies. This turns out to be surprisingly difficult.

If we had a complete list of consistent time-dependent solutions, then we would face the same sort of question that we posed earlier in the particle physics context. What is the principle by which the particular one that describes the evolution of our Universe is selected? Was there a pre-big-bang era, and how did the Universe begin? Can string theory provide an understanding of the early period of exponential growth (inflation) that occurred in the very early Universe? How much of the observed large-scale structure of the Universe can be deduced from first principles?

We may not be close to answering these questions, but something important is happening. The field of "superstring cosmology" is emerging as new and respectable discipline. String theorists and string theory considerations are injecting new ideas into the study of cosmology. This might be the arena in which predictions that are specific to string theory first confront data.

5.6 Find the String Theory Dual of QCD

We pointed out in the beginning of this article that string theory originated in an attempt to describe the strong interactions, and that most workers abandoned the subject when it was realized that QCD is the correct theory. Nonetheless, it is probably true that there is a string theory dual to QCD that provides an alternative, but equivalent, description of the strong interactions. Finding it would be enormously useful and informative. Presumably, this hasn't been achieved yet because this string theory is actually more difficult to construct than the "critical" gravitational string theories that we have been discussing. Knowing this string theory would make it possible to compute properties of hadrons analytically to good approximation, something that is extremely difficult, even using state-of-the-art computers, starting from QCD.

One lesson of AdS/CFT is that the string dual of QCD should not be four-dimensional. As emphasized by Polyakov, it should, at the least, possess a fifth dimension that is dual to scale size [37]. Finding this string theory is a tough problem, but there is a reasonable chance that it will meet with success.

5.7 Develop Mathematical Tools and Concepts

String theory is up against the frontiers of most major branches of mathematics. Given the experience to date, there is little doubt that future developments in string theory will utilize many mathematical tools and concepts that do not currently exist. If recent trends continue, some of these will be developed in response to the needs of string theory, while others will be invented independently by mathematicians and then utilized by string theorists. The need for cutting edge mathematics is promoting a very healthy relationship between large segments of the theoretical physics and mathematics communities. Not only are fundamental forces being unified, but so are disciplines.

Acknowledgments

This work was supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.

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