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Black Holes and Holography in String Theory

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Abstract.

Black holes are very puzzling objects that are predicted by general relativity. Once we combine general relativity and quantum mechanics we find that black holes should be thermal objects. This thermal description gives some hints about the microscopic constituents of spacetime and leads to the concept of holography. Namely, that physics inside a region of spacetime can be described in terms of a theory on the boundary.

In string theory a very precise description was found for certain negatively curved spacetimes. The theory at the boundary is a simple theory of interacting particles of the kind that we are very familiar from ordinary particle physics. This has interesting implications for the quantum mechanical description of black holes.

1 Black holes and holography in string theory

1.1 Black holes

Black holes are one of the most fascinating objects predicted by Einstein's theory of general relativity. Black holes have an interesting history and they have been a source of many theoretical surprises that have led to a better understanding of the nature of spacetime.

General relativity is a theory that describes spacetime. According to this theory spacetime is not flat and fixed, it is curved and it is dynamical. Its shape depends on the matter that moves on it and the matter motion depends on the shape of spacetime. Physicists always try to study the simplest situations first. So in 1916, shortly after general relativity was invented, a young German, called Karl Schwarzschild, found the simplest spherically symmetric solution of Einstein's equations. These equations describe a particular geometry which was thought to be the geometry generated by a point-like mass. Instead of saying what the geometry is, let us concentrate on one of its features: the rate at which stationary clocks tic. Here, on the surface of the earth, a clock on the top floor of a building runs faster than one in the bottom. This is a measurable effect but it is very small: of the order of one part in 10^{15}. For more massive objects the effect is more important. A clock at the surface of the sun runs slower by one part in a million. A clock on the surface of a neutron star runs at 70 % of the speed of a clock far away from the star. In this case we see that it is a large effect. The solution that Schwarzschild found indicated that a clock at the "center" would completely stop. At first most physicists thought that this was an unphysical result, a product of an overly simplified analysis.

Further studies showed that the "center" of Schwarzschild's solution is, in fact, a completely smooth surface. An observer that is traveling through spacetime could go through this region without feeling anything strange or peculiar. The people who stay outside the black hole see that all signals coming from the falling observer slow down until they eventually die out for all practical purposes. The surface where stationary clocks slow down to zero is called a "horizon". This surface marks a point of no return. An observer who crosses this surface will not be able to come back out again and will crush into a "singularity" in the interior. The singularity is a region of very high spacetime curvature that will rip him apart. For a black hole of the mass of the earth the horizon would have a size of $1cm$, for a black hole of the mass of the sun it would be $3Km$.

Black holes can form in astrophysical processes when stars that are a few times more massive than the sun run out of their nuclear fuel and implode under their gravitational force. There is a great deal of observational evidence that there are some black holes out there in the universe.

The focus of this talk is not to describe astrophysical black holes but to explore the implications of black holes for the structure of spacetime.

According to Einstein's theory a black hole is a hole in spacetime, once you fall in you cannot come back again. Whatever is thrown into a black hole is forever lost.

The next surprise that black holes had in store for us came about when quantum effects were studied. In quantum mechanics the vacuum is not merely the absence of particles. The vacuum is a very interesting state where all the time we have particle pairs being created and destroyed. In flat space we have no net production of particles since energy has to be conserved. All particles that are produced have to annihilate very quickly. Steven Hawking showed that when a horizon is present this is no longer the case. What can happen is that a particle with positive energy and one with negative energy are created in the vicinity of the horizon. The negative energy particle falls into the black hole and the positive energy one flies away. In flat space this is not possible because we cannot have negative energy particles. However, on the other side of the horizon a particle that has negative energy from the point of view of an observer far away can have positive energy from the point of view of an observer inside the horizon. The net effect is that the black hole emits particles. The emitted particles have a thermal distribution, with a temperature that is inversely proportional to the black hole mass. For solar-mass black holes this temperature is too tiny for this effect to be measurable. If the black hole were in empty space it would slowly loose mass and become smaller. Smaller mass black holes could have higher temperatures. A black hole with a mass of the order of $10^{18} Kg$ (the mass of a mountain range) would have a temperature of a thousand degrees and would emit light as a 1 milli-Watt light bulb. As its mass becomes smaller and smaller its temperature would rise and it would evaporate faster and faster until it, presumably, evaporates completely.

In ordinary physics, thermal properties always arise from the motion of the constituents. For example, the temperature of the air is related to the average speed of the air molecules. There is a closely related concept, called Entropy. The entropy is the amount of disorder associated to the motion of all the constituents. The entropy is related to the temperature by the laws of thermodynamics, so it can be computed without knowing the microscopic details of the system. Hawking and Bekenstein showed that the entropy of a black hole is the same as the area of the horizon divided by the square of the Planck length, where $l_{Planck} = 10^{-33} cm$. For a macroscopic black hole this is an enormous entropy. It turns out that the laws of thermodynamics continue to be valid if the black hole contribution to the entropy is included. These are extremely puzzling results since it is not at all clear what the "constituents" of a black hole really are. The black hole is a hole in spacetime, so finding its constituents is intimately related to finding the most fundamental constituents of spacetime geometry.

It is very interesting that the entropy of a black hole is proportional to its area and not its volume. In fact, its has been proposed that in a theory that includes quantum mechanics and gravity the number of constituents that are necessary to describe a system cannot be bigger than the area of a surface that encloses it [1]. This implies that spacetime is rather different from an ordinary solid since in the latter case, the number of constituents (the atoms) grows like the volume. For most practical purposes this entropy bound is not very stringent, but it has interesting theoretical implications, since it suggests that a region of spacetime can be described in terms of constituents that live on the boundary of this region.

In the remainder of this article we will explore some of these issues in the context of string theory. We will introduce some interesting curved spacetimes which can be described completely in terms of an ordinary theory of particles living on the boundary. We will first describe these spacetimes in some detail, we will then describe some properties of the particle theory on the boundary and then some aspects of the relation between the two. Black holes can be understood as simple thermal configurations of particles on the boundary.

2 Negatively curved spacetimes

All of us are familiar with Euclidean geometry, where space is flat. We are also familiar with some curved spaces. For example, the simplest possible space with positive curvature is the surface of a

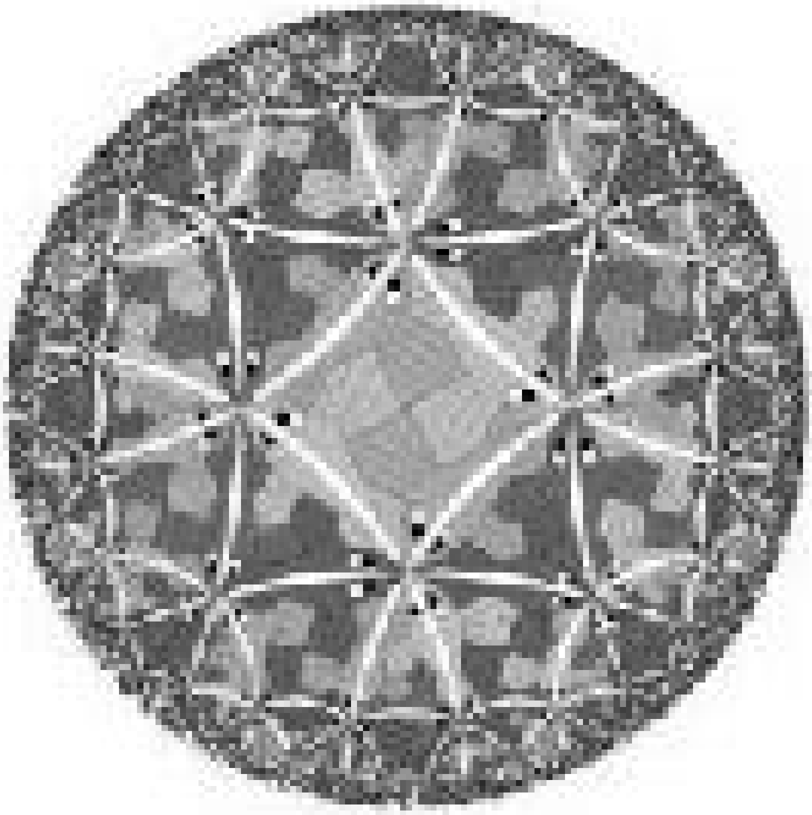

Figure 1: This drawing by Escher tries to capture the geometry of hyperbolic space. It is a projection of hyperbolic space onto a disk. Each figure has the same proper size, that is to say in the original hyperbolic space they all look the same size but due to the distorting effects of the projection they get smaller as we go to the boundary of the disk. In fact the boundary of the disk is an infinite distance away from any point in the interior. A similar distortion is present when we represent a world map on a plane. With the standard projection the region near the poles look disproportionately big on the map. In this projection of hyperbolic space we have the opposite effect. Hyperbolic space is infinite in size but it looks finite in the drawing because the region near the boundary has been re-scaled by a very large factor.

sphere. The simplest space with negative curvature is called hyperbolic space. This might not be as familiar, but it has fascinated scientists and artists alike. The Dutch painter Escher has given us some nice representations of hyperbolic space, one of which can be seen in Fig 1. In hyperbolic space the length of a circle of radius R is bigger than $2\pi R$ (in flat space it is equal to $2\pi R$, and for a sphere is smaller that $2\pi R$). Whereas a sphere has finite volume, hyperbolic space has infinite volume.

Once we include time into the game we can similarly consider space-*times* with positive or negative curvature. The simplest space-time with positive curvature is called de-Sitter space (de Sitter was the Dutch physicist who introduced it). Many cosmologists believe that the very early universe was very close to de Sitter space. On the other hand, the simplest negatively curved space-time is called anti-de-Sitter space. It is similar to hyperbolic space except that it now contains a time direction. The spatial geometry at a constant moment in time is again that of hyperbolic space. The space-time looks the same at all times, it is neither expanding nor contracting. If one were to be freely floating in anti-de-Sitter space one would feel as if one were at the bottom of a gravitational potential well. Any object that one throws out will come back. Surprisingly the time

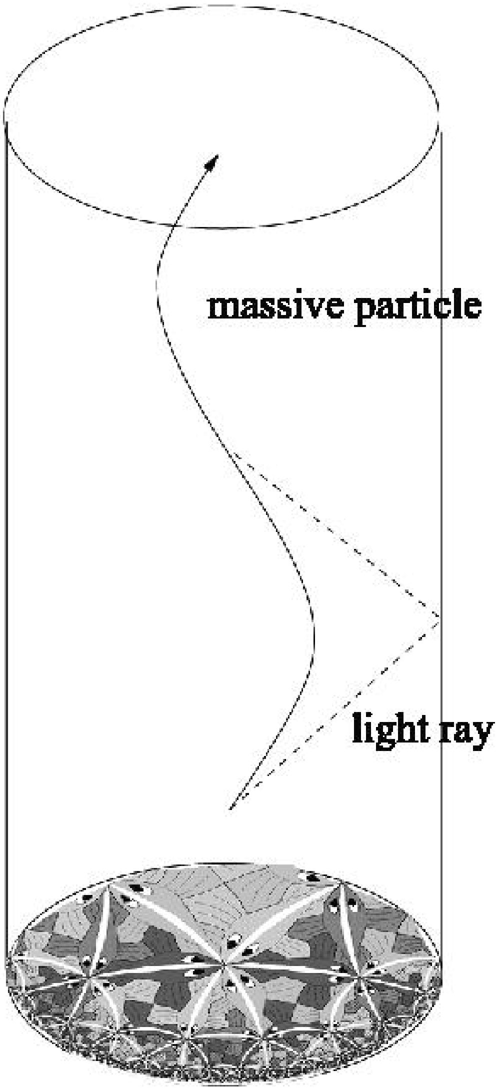

Figure 2: This is a figure of Anti-de-Sitter space-time. It looks like a cylinder with time running in the vertical direction. Each spatial cross section looks like hyperbolic space (see figure one). The solid wavy line represents the trajectory of a massive particle. We see that it oscillates around the center of the space, so an observer at the center would say that gravitational forces pull everything towards the center. The dashed line represents the trajectory of a light ray. It goes to the boundary and back in finite time from the point of view of an observer who stays at the origin.

it takes for an object to come back is independent of how fast one throws it. So if we throw a particle extremely fast it can reach arbitrarily distant points and come back in finite time. If we send a light ray, which consist of particles moving at the maximum possible speed (the speed of light), it will reach infinity and come back in finite time. It is as if there was a mirror at infinity which reflects light back in. Note that if the curvature is very small then we would feel more or less as we feel in flat space. In some sense we can draw an Escher-like picture anti-de-Sitter space as a solid cylinder with time running along the cylinder. See Fig 2. Again we are making a projection that maps the infinite spatial sections into finite disks as Escher did for hyperbolic space. Anti-de-Sitter spacetime does not look like our universe, which is expanding, but it is nevertheless an interesting spacetime to consider. For Anti-de-Sitter spacetimes one can give a complete quantum gravity description of physics in the interior.

3 A precise description of quantum gravity

Anti-de-Sitter space-times received a lot of attention from the string theory community due to the conjecture that quantum gravity in Anti-de-Sitter space-times is equivalent to an ordinary particle theory [2, 3]. Subsequently many researchers contributed to the exploration and generalization of this conjecture, providing mounting evidence that it is correct. If this conjecture is correct, we can use a particle theory to define a gravity theory. The definition is a bit indirect because the particles defining the gravity theory do not move on anti-de-Sitter space-time, they move on its boundary. We can think of this boundary as the surface at a very large radial distance from the center. This surface contains two spatial dimensions, which form an ordinary sphere, and a time dimension. The boundary has one dimension less than the interior, i.e. a four dimensional anti-de-Sitter space has a boundary which has 2 spatial dimensions and one time dimension. So we start with a theory of quantum particles moving on a fixed classical space-time which looks like the boundary of anti-de-Sitter space, namely a sphere and the time direction. These particles interact strongly with each other. These strong interactions imply that they form bound states and these bound states behave as if they were moving in anti-de-Sitter space-time. Note that despite the fact that the particles that make the bound state move on the boundary the bound state itself can be thought of as living inside. This looks very surprising at first sight. In order to specify a point in the interior we need one more coordinate than to specify a point on the boundary. The main observation is that the particle theory that lives on the boundary is scale invariant. This implies that these bound states can have any possible size. So in order to completely specify the state we need to give its position on the boundary theory and, in addition, we need to specify its size. This bound state will correspond to a particle in the bulk anti-de-Sitter spacetime. The size of the bound state in the boundary theory is related to the distance of the bulk particle to the boundary. The bulk particle has a fixed size. This is represented in Fig 3. Particles in the interior that are separated in the radial direction do not interact very much with each other. Correspondingly, states with different sizes do not interact very much in the boundary theory, even if they are on top of each other. In fact one of these states behaves precisely as a "graviton". Since we know precisely the boundary theory we know precisely how these gravitons interact with each other. It turns out that the boundary theory is such that the interactions of these bound states at low energies are the same as those of gravitons in Einstein's theory. In this description, gravity in four dimensions is an "emergent" phenomenon arising from the interactions of a theory in three dimensions which does not contain gravity.

Note that in the end we are describing everything that happens in the interior of Anti-de-Sitter by a theory living on the boundary. In fact, a theory of quantum gravity is expected to have this property, called "holography". An optical hologram encodes the three dimensional shape of an object on a two dimensional photographic plate. In a similar way, "holography" postulates that one should be able to describe the interior of a space-time region by a theory living on its boundary [1].

4 The boundary theory

It is well known that a system of interacting particles can produce effects which are not obvious from the point of view of the original particles. All the richness of the physical phenomena that we see in our everyday life is due to particle interactions, with no interactions we would have a very boring world. The particles living at the boundary are very similar to the particles in terms of which we describe strong interactions. The strong force is due to the exchange of a particle called a "gluon". A gluon is rather similar to the photon, the particle responsible for electromagnetic interactions. While the photon can only interact with one type of charge (the electric charge) the gluon can interact with many types of charges, which physicists call "colors". We call these "colors" because in the theory of strong interactions we have three of them, the same as the number of primary colors. From the theoretical point of view we can consider theories with a very large number of colors. In fact, the radius of curvature of the anti-de-Sitter space is proportional to the number of colors. In order to have an astronomically large anti-de-Sitter space we need to have an

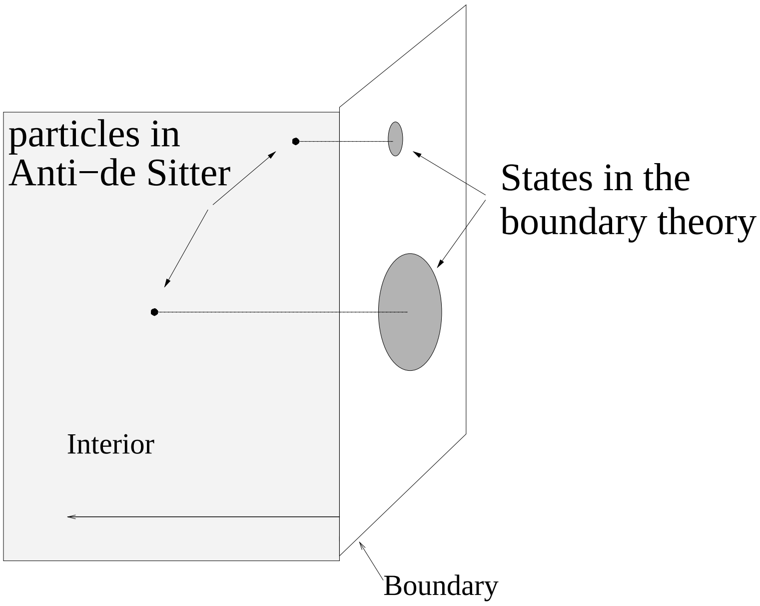

Figure 3: We see a pair of bound states in the boundary theory. In the boundary theory these bound states can come in different sizes. The bigger state corresponds to a particle in the interior that is further from the boundary that the particle that corresponds to the smaller state. Though the states have different sizes in the boundary theory, they have the same size in the interior but they are located at different distances from the boundary. The gravitational force in the interior wants to move the particle away from the boundary. This corresponds, in the boundary theory, to the fact that the bound states will want to become larger.

astronomically large number of colors, of the order of 10^{60}. The precise theory at the boundary depends on the precise theory in the interior of the space-time. For example, we can have only gravitational forces in the interior, or gravity plus some extra force like the electromagnetic force, etc. Different extra forces in the interior are associated to different boundary theories which differ in the detailed way that the boundary particles interact with each other. We do not have a general prescription for finding the associated boundary theory, but we have many examples where we can say precisely what the interior theory is and what the boundary theory is. The cases that are best understood are those when both of the theories have a high degree of symmetry. Unfortunately we do not yet have a construction of a boundary theory associated to a theory in the interior with the three other forces that we have in our universe. The conjecture, however, lets us understand aspects of quantum gravity that do not depend on the details of the other forces.

It is very surprising that a particle theory on the boundary can dynamically give rise to physics in one more dimension. We are starting with a particle theory on a sphere (and one time dimension) without gravity. Its dynamics can be such that the resulting physics is best described in terms of a gravitational theory in four space-time dimensions. The interactions are responsible for manufacturing one more dimension. In fact, it was long suspected that interacting gluons with a large number of colors give rise to a string theory, where the strings are made out of these interacting gluons [4]. It was then understood that these strings move in one more dimension [5]. These strings are actually moving in the interior of the negatively curved space-time. So the quantum gravity in the interior is a string theory. In string theory the graviton is a tiny string. String theory says precisely how these strings interact and since the graviton is one such string we get then a theory of quantum gravity.

This relationship between the gluons that live on the boundary and the gravity theory in the interior is very useful for doing complicated calculations in the boundary theory. Calculations that are very complicated in the boundary theory become very simple in terms of the theory in the interior. For example, one of the most perplexing properties of strong interactions is confinement.

Confinement is the reason that we do not observe free quarks. When we separate two quarks a string of gluons forms that joins them. ¿From the point of view of the string theory living in the bulk this is a string that ends at two points on the boundary. This string pulls the quarks together and in order to separate them by a very long distance we need a lot of energy because we have to stretch the string that joins them.

5 Shedding some light on black holes

The elementary quantum excitation of space-time is a graviton. A black hole contains a large number of strongly interacting gravitons, they interact so much that they substantially alter the shape of space-time. If the black hole is in anti-de-Sitter space-time we can use the conjectured relation to the particle theory on the boundary to describe it. The black hole is some configuration of particles on the boundary. The number of particles is very large and they are all moving around. The temperature is then related to how fast these particles are moving around in the boundary theory. So the temperature on the boundary theory, computed according to the usual rules of statistical mechanics, is the same as the black hole temperature computed by Hawking. These thermal aspects of black holes had raised some doubts about the existence of a quantum mechanical theory of gravity. The description in terms of the particle theory at the boundary obeys the ordinary rules of quantum mechanics. The boundary theory has the ability of describing black holes in a way that is consistent with quantum mechanics. In summary, we now have theories of quantum gravity that describe black holes exactly and in all detail. Of course, doing the computations necessary to answer specific questions can be a daunting task, but a task that can in principle be done.

6 Looking at the future

We have seen that gravity in four dimensional anti-de-Sitter appears as an emergent property of a system of particles moving on a sphere. A crucial aspect of these negatively curved space-times is that they have a boundary where time is well defined. The boundary has existed and will exist for ever. Ordinary quantum mechanical systems require an absolute notion of time. In our case, this is provided by the time in the boundary theory. This description is useful if we want to describe black holes as seen from the outside. It is not obviously useful if we want to describe the interior of black holes, though the physics of the interior is probably contained in the boundary theory. Another open question is how to describe expanding universes which come from a Big Bang. One important lesson that one can draw from this conjecture is that quantum gravity can be very simple, when viewed in terms of the appropriate variables. Let's hope we will soon find a simple description for the Big Bang!

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