

Dynamical Consequences of Strong Entanglement

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Abstract. Motion in quantum theory as a unitary evolution has very different properties compared to classical physics. If the phase relations defining unitary dynamics are destroyed or unavailable, motion becomes impossible (Zeno effect). The most important mechanism is decoherence, arising from coupling of a quantum system to its environment. Macroscopic systems are not frozen, although strong decoherence is important to understand classical objects within the quantum framework. These two conflicting consequences of strong decoherence are analyzed and compared.

1 Introduction

It seems to be widely accepted by now that non-classical states of macroscopic objects can never show up in the laboratory or elsewhere since they are unstable against decoherence. This explains superselection rules, that is, kinematical restrictions in the space of possible (quantum) states. The observation that macroscopic objects are under “continuous observation” by their natural environment paved the way for our current understanding of the quantum-to-classical transition [1].

Since in a consistent quantum treatment macro-objects are obviously to be considered as open systems, their dynamics can longer follow a Schrödinger equation. This alone invalidates the textbook derivations of the classical limit via Ehrenfest theorems. Instead, one has to study the consequences of strong measurement-like interaction of the considered system with its environment. The resulting entanglement not only superselects certain states, which are then called “classical” by definition, but also leads to dynamical consequences. Very simple arguments seem to show that strong decoherence, that is, strong entanglement, leads to slowing down of the dynamics of *any* system. However, the objects in our macroscopic world obviously *are* moving around and there seems to be no “Zeno effect”. How this puzzle can be solved will be discussed in the following sections.

2 The quantum Zeno effect

The quantum Zeno effect was discovered independently by several authors when studying the properties of decay probabilities in quantum theory. The now popular term “quantum Zeno effect” was introduced by Misra and Sudarshan [2].

Let a system be described by some “undecayed” state $|\Psi(0)\rangle = |u\rangle$ at some initial time $t = 0$. The probability $P(t)$ to find it again in this “undecayed” state at a later time t is

$$P(t) = |\langle u | e^{-iHt} | u \rangle|^2 \quad (1)$$

where H is the Hamiltonian of the system. For small times we can expand $P(t)$, yielding

$$P(t) = 1 - (\Delta H)^2 t^2 + O(t^4) \quad (2)$$

with

$$(\Delta H)^2 = \langle u | H^2 | u \rangle - \langle u | H | u \rangle^2. \quad (3)$$

The important feature to notice here is the *quadratic* time dependence of the survival probability. This may be compared with usual exponential decay law

$$P(t) = \exp(-\Gamma t), \quad (4)$$

which leads to a *linear* time dependence for small times,

$$P(t) = 1 - \Gamma t + \dots \quad (5)$$

This raises the question, how these two differing results can be made compatible. Both look fundamental, but they obviously contradict each other. This conflict can be made even stronger, when we consider the case of repeated measurements in a short time interval.

Suppose we repeat the measurement N times during the interval $[0, t]$. Then the non-decay (survival) probability is

$$P_N(t) \approx \left[1 - (\Delta H)^2 \left(\frac{t}{N} \right)^2 \right]^N > P(t), \quad (6)$$

which for large N gives

$$P_N(t) = 1 - (\Delta H)^2 \frac{t^2}{N} + \dots \xrightarrow{N \rightarrow \infty} 1. \quad (7)$$

This is the Zeno effect: Sufficiently dense measurements should halt any motion!

There is no Zeno effect, if the system decays according to the exponential decay law, since in this case trivially

$$P_N(t) = \left(\exp \left(-\Gamma \frac{t}{N} \right) \right)^N = \exp(-\Gamma t) = P(t). \quad (8)$$

The conclusion is, that any system showing a quadratic short-time behavior is very sensitive to measurements, whereas an exponentially decaying system does not care about whether its decay status is measured or not, that is, it behaves classically in this respect.

If a system is governed by the Schrödinger equation, as used in Equ. (1), the transition probability for small times *must* start quadratically, hence the exponential decay law can only be an approximation for larger times.¹ What happens in the limit of “continuous” observation? The Zeno argument seems to show that there will be no motion at all!

To gain a better understanding of what is going on here, I will discuss in the following why motion is slowed down by measurements. In addition, the measurement process itself will be described by a unitary evolution following the Schrödinger equation as the fundamental law of motion for quantum states. It will turn out, that the Zeno effect can be understood as a dynamical process and the collapse of the wave function is not required.

3 Interference, Motion and Measurement in Quantum Theory

Why does measurement slow down motion in quantum theory, but not in classical physics? The reason can be traced back to the very nature of quantum evolution. Quantum dynamics is unitary and can be viewed as a rotation in Hilbert space, see Fig. 1. If a system is going to move from a state $|a\rangle$ to another state $|b\rangle$, it has to go through a superposition state $|a\rangle + |b\rangle$. An essential feature of such a superposition is the presence of interference (coherence). As is well known, such a superposition has properties which none of its components has – it is an entirely new state.² Unitary evolution from $|a\rangle$ to $|b\rangle$ requires all the phase relations contained in the intermediate state $|a\rangle + |b\rangle$. Phase relations are destroyed by measurements, so it is not surprising that motion becomes impossible in quantum theory if coherence is completely absent!

As an example consider the evolution of a two-state system from an initial state $|1\rangle$ as a two-step process connecting times $0, t,$ and $2t$, as shown in Fig. 2. If a_{ij} are transition amplitudes

¹There is a certain irony in this situation, since – at least in popular accounts – exponential (“random”) decay is used as a major argument that classical physics has to be replaced by a new (quantum) theory. But there is no strict exponential decay law in quantum theory.

²This is the reason why stochastic models for quantum evolution are unsuccessful: A superposition cannot be replaced by an ensemble of its components.

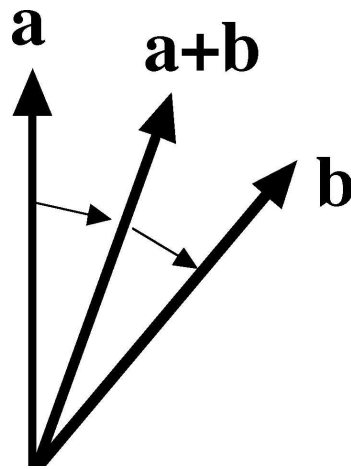


Figure 1: Evolution in quantum theory can be viewed as a rotation connecting an initial state $|a\rangle$ with a final state $|b\rangle$. For intermediate times a superposition $|a\rangle + |b\rangle$ is required for undisturbed motion.

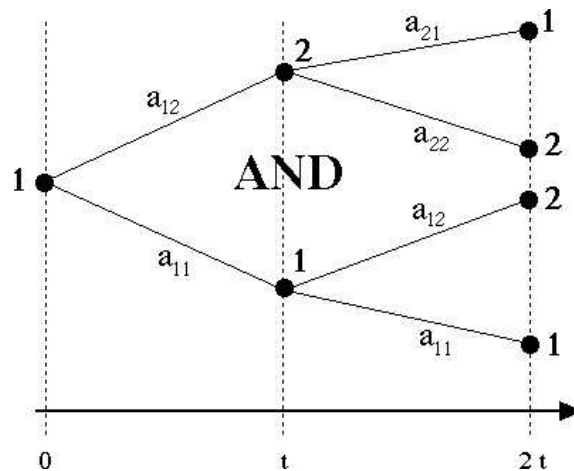


Figure 2: Evolution of a two-state system away from initial state $|1\rangle$. The amplitude (and therefore the probability) of state $|2\rangle$ at time $2t$ depends on the phases contained in the superposition of $|1\rangle$ and $|2\rangle$ at the intermediate time t , as in a double-slit experiment.

(calculated from the Schrödinger equation) we have the chain

$$\begin{aligned}
 t = 0 : & \quad |1\rangle \\
 & \longrightarrow a_{11} |1\rangle + a_{12} |2\rangle \\
 & \longrightarrow (a_{11}^2 + a_{12}a_{21}) |1\rangle + (a_{12}a_{22} + a_{11}a_{12}) |2\rangle.
 \end{aligned}
 \tag{9}$$

The final probability for state $|2\rangle$ at time $2t$ is then

$$\begin{aligned}
 P_2 &= |a_{12}a_{22} + a_{11}a_{12}|^2 \\
 &\approx V^2(2t)^2
 \end{aligned}
 \tag{10}$$

with

$$V = \langle 1 | H | 2 \rangle.
 \tag{11}$$

Clearly the value for P_2 depends essentially on the presence of interference terms. In a sense unitary evolution is an ongoing double- (or multi-)slit experiment!³

³Obviously, the above model is nothing more than a very primitive version of the path-integral formalism.

Now compare this evolution with the same process, but a measurement is made at the intermediate time t . This measurement may be described by a collapse producing an ensemble (that is, resulting in $|1\rangle$ or $|2\rangle$), or dynamically by coupling to another degree of freedom. In the latter case

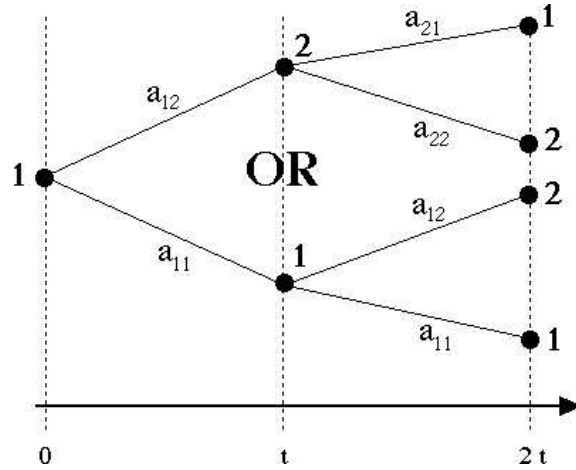


Figure 3: Evolution of a two-state system with measurement. The probability for state $|2\rangle$ at time $2t$ results solely from the transition probabilities to intermediate states at time t . The loss of phase relations leads to a decrease of the total transition probability.

an entangled state containing the system and the measuring device $|\Phi\rangle$ (or, generally, the system's environment) ensues (more on this in the next section). The equations now look like

$$\begin{aligned}
 t = 0 : & \quad |1\rangle |\Phi\rangle \\
 & \longrightarrow (a_{11} |1\rangle + a_{12} |2\rangle) |\Phi\rangle \\
 & \longrightarrow a_{11} |1\rangle |\Phi_1\rangle + a_{12} |2\rangle |\Phi_2\rangle \\
 & \longrightarrow (a_{11}^2 |\Phi_1\rangle + a_{12} a_{21} |\Phi_2\rangle) |1\rangle + (a_{12} a_{22} |\Phi_1\rangle + a_{11} a_{12} |\Phi_2\rangle) |2\rangle
 \end{aligned} \tag{12}$$

(the third line describes the new measurement step) and the transition probability is given by

$$\begin{aligned}
 P_2 &= |a_{12} a_{22}|^2 + |a_{11} a_{12}|^2 \\
 &\approx \frac{1}{2} V^2 (2t)^2.
 \end{aligned} \tag{13}$$

Since the interference terms are missing, we lose half of the probability! Clearly then, if we divide the time interval not in two but into N steps, the transition probability is reduced by a factor $1/N$: the Zeno effect. This reduction is a sole consequence of entanglement, without any “disturbance” of the measured system, since the measurement is assumed ideal in this model. No coherence, no motion!

The Zeno effect can also be seen more formally from the von Neumann equation for the density matrix. If coherence is absent in a certain basis, the density matrix is diagonal, i.e.,

$$\rho_{nm} = \rho_{nn} \delta_{nm}. \tag{14}$$

But then no evolution is possible, since the von Neumann equation immediately yields

$$\frac{d}{dt} \rho_{nn} = \sum_k (H_{nk} \rho_{kn} - \rho_{nk} H_{kn}) \equiv 0. \tag{15}$$

4 Measurement as a Dynamical Process: Decoherence

To further analyze the Zeno effect I will consider a specific model for measurements of an N -state system. As a preparation, let me shortly review the dynamical description of a measurement

process. In a dynamical description of measurement, the well-known loss of interference during measurement follows from a certain kind of interaction between a system and its environment.

Following von Neumann, consider an interaction between an N -state system and a “measurement device” in the form

$$|n\rangle |\Phi_0\rangle \longrightarrow \exp(-iHT) |n\rangle |\Phi_0\rangle = |n\rangle |\Phi_n\rangle \quad (16)$$

where $|n\rangle$ are the system states to be discriminated by the measurement device and $|\Phi_n\rangle$ are “pointer states” telling which state of the system has been found. H is an appropriate interaction leading after the completion of the measurement (at time T) to orthogonal states of the measuring device. Since in Equ. (16) the system state is not changed, this measurement is called “ideal” (recoil-free). A general initial state of the system will – via the superposition principle – lead to an entangled state,

$$\left(\sum_n c_n |n\rangle \right) |\Phi_0\rangle \longrightarrow \sum_n c_n |n\rangle |\Phi_n\rangle. \quad (17)$$

This correlated state is still pure and does therefore *not* represent an ensemble of measurement results (therefore such a model alone does not solve the measurement problem of quantum theory). The important point is that the phase relations between different n are *delocalized* into the larger system and are no longer available at the system alone. Therefore the system *appears* to be in one of the states $|n\rangle$, formally described by the diagonalization of its density matrix,

$$\begin{aligned} \rho &= \sum_{n,m} c_m^* c_n |n\rangle \langle m| \\ &\longrightarrow \sum_{n,m} c_m^* c_n \langle \Phi_m | \Phi_n \rangle |n\rangle \langle m| \\ &= \sum_n |c_n|^2 |n\rangle \langle n|, \end{aligned} \quad (18)$$

where the last line is valid, if the pointer (or environmental) states are orthogonal, $\langle \Phi_m | \Phi_n \rangle = 0$.

Any measurement-like interaction will therefore produce an apparent ensemble of system states. This process is now usually called “decoherence” [1]. Note that the origin of this effect is *not* a disturbance of the system. Quite to the contrary: the system states $|n\rangle$ remain unchanged, but they “disturb” (change) the environment!

5 Strong Decoherence of a Two-State System

As a first application of the von-Neumann measurement model let us look at an explicit scheme for a two-state system with Hamiltonian

$$\begin{aligned} H &= H_0 + H_{\text{int}} \\ &= V(|1\rangle \langle 2| + |2\rangle \langle 1|) + E |2\rangle \langle 2| \\ &\quad + \gamma \hat{p} (|1\rangle \langle 1| - |2\rangle \langle 2|). \end{aligned} \quad (19)$$

Here the momentum operator \hat{p} leads to a shift of a pointer wavefunction $\Phi(x)$ “to the right” or “to the left”, depending on the state of the measured system, γ represents a measure of the strength of this interaction. Because of the structure of the Hamiltonian, this interaction is recoil-free. This model can be solved exactly and shows the expected damped oscillations. In view of the Zeno effect we are mostly interested in the limit of strong coupling. Here the solutions (calculated in perturbation theory) show two interesting features, as displayed in Figs. 4 and 5 [3]. First, the transition probability from $|1\rangle$ to $|2\rangle$ depends in a complicated way on the coupling strength, but for large coupling always decreases with increasing interaction. This is the expected Zeno behavior.

If we look at the time dependence of the transition probability, we can see the quadratic behavior for very small times (as is required by the general theorem Equ. (2)), but soon the

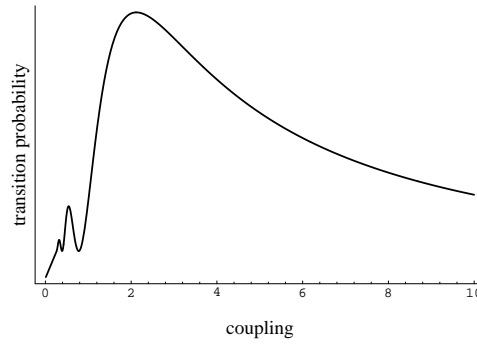


Figure 4: Transition probability as a function of the coupling strength in a two-state model. For strong coupling, transitions are always damped (Zeno effect).

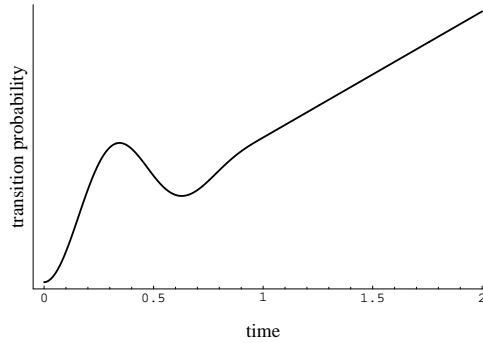


Figure 5: Transition probability as a function of time. If the measurement can be considered complete (here at $t \approx 1$), the transition probability grows linearly (constant transition rates)

transition probability grows linearly, as in an exponentially decaying system (the rate, however, still depends on the coupling strength).

A realization of the quantum Zeno effect has been achieved in an experiment [4] where the two-state system is represented in the form of an atomic transition and the measurement process is realized by coupling to a third state which emits fluorescence radiation, see. Fig. 6.

The Zeno effect also shows up in a curious way in a recent proposal of “interaction-free measurement”.

Early ideas about “negative result” or “interaction-free” measurements [5] can be combined with the Zeno mechanism [6]. One of these schemes is exemplified in Fig. 7. If a horizontally polarized photon is sent through N polarization rotators (or repeatedly through the same one) each of which rotates the polarization by an angle $\Delta\Theta = \frac{\pi}{2N}$, the photon ends up with vertical polarization, that is the probability to find horizontal polarization is zero,

$$P_H = 0. \quad (20)$$

If this evolution is interrupted by a horizontal polarizer (absorber) the probability of transmission is (similar to Eqs. (6) and (7)) given by

$$P'_H = \cos^{2N} \Delta\Theta = \cos^{2N} \frac{\pi}{2N} \approx 1 - \frac{\pi^2}{4N} \longrightarrow 1. \quad (21)$$

To implement this idea, a photon is injected into the setup shown in Fig. 7 and goes N times through the rectangular path, as indicated. The initial polarization is rotated at R by an angle $\Delta\Theta = \frac{\pi}{2N}$ on each passage. In the absence of the absorbing object, the polarizing beam splitters, making up a Mach-Zehnder interferometer, are adjusted to have no effect. That is, the vertical component V is coherently recombined with the horizontal one (H) at the second beamsplitter

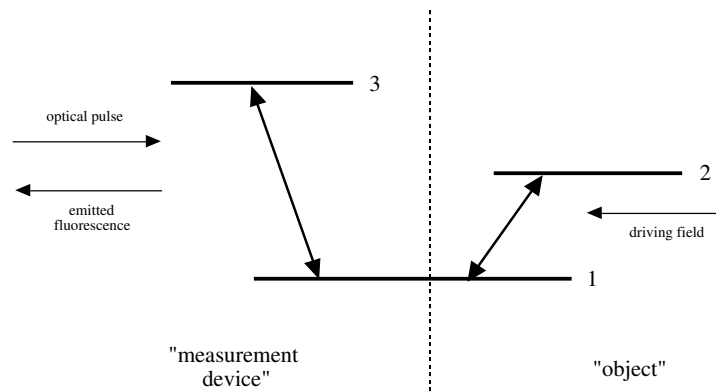


Figure 6: Zeno experiment in atomic physics. The two-state system under repeated observation is represented by a transition between states $|1\rangle$ and $|2\rangle$. Measurement is accomplished through an optical pulse leading to fluorescence from level $|3\rangle$, if the state $|1\rangle$ is present.

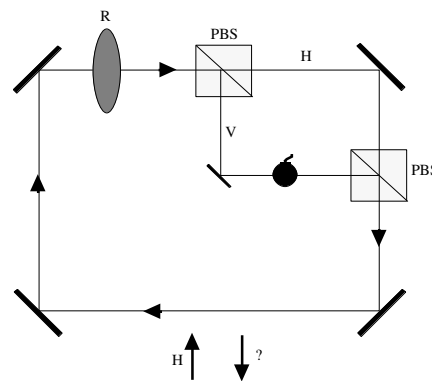


Figure 7: Scheme of “interaction-free interrogation” as a variant of the Zeno effect. Without the absorbing object (the bomb), the polarization of the injected photon (initially horizontal) is rotated by the rotator R by a small angle on every passage. The two polarizing beam splitters PBS have no effect, if properly adjusted, since horizontal and vertical components are recombined coherently. If an absorbing object is present, the vertical polarization is removed at every passage. Inspecting the photon after many cycles allows one to infer the presence of the object with high probability, while the photon is only very infrequently absorbed.

to reproduce the rotated state of polarization. If, however, the “bomb” is present, the vertical component is absorbed at each step. After N cycles, the photon is now still horizontally polarized, thereby indicating the presence of the object with probability near one, or has been absorbed (with arbitrarily small probability). For details of experimental setups see [7].

One should be aware of the fact, that the term “interaction-free” is seriously misleading, since the Zeno mechanism is a consequence of *strong* interaction. Part of this conceptual confusion is related to the classical particle pictures often used in the interpretation of interference experiments, in particular “negative-result measurements”.

6 Strong Decoherence of Many-State Systems

Why does the Zeno effect not show up in our macroscopic world? I will consider two examples of classical dynamics. The first is the motion of a massive object such as a dust particle or a planet. The second example will be a reconsideration of Pauli’s rate equation, describing classical random

processes, where interference apparently plays no role. In both cases it will turn out that (1) continuous measurement (i.e. decoherence) is an essential ingredient for deriving classical motion and (2) the Zeno effect plays no role.

6.1 Macroscopic objects

With hindsight it seems to be trivial observation that all macroscopic objects are strongly interacting with their natural environment. The consequences have been analyzed only rather late in the history of quantum theory [8, 9]. One reason for this is certainly the prevailing Copenhagen orthodoxy. For generations students were told, that quantum theory should only be used for microscopic objects, while macroscopic bodies are necessarily described by classical physics.

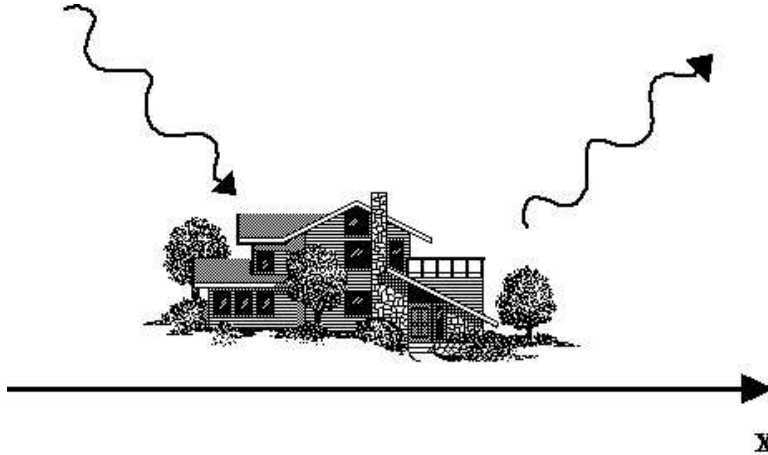


Figure 8: Macroscopic objects can never be considered as isolated from their natural environment. Irreversible scattering processes lead to ever-increasing entanglement.

The typical scenario is represented by scattering processes, where the state of the scattered objects, a photon or a molecule typically depends on the position of the macroscopic body. Quantitative estimates [9] show a strong effect, even in extreme situations, for example, a dust particle scattering only cosmic background radiation. For small distances, interference is damped according to

$$\rho(x, x', t) = \rho(x, x', 0) \exp[-\Lambda t(x - x')^2] \quad (22)$$

with a “localization rate” Λ given by

$$\Lambda = \frac{k^2 N v \sigma_{eff}}{V} \quad (23)$$

Here k is the wave vector of the scattered particle, Nv/V the incoming flux and σ_{eff} of the order of the total cross section. Some typical numbers are shown in the table.

The equation of motion of, say, a dust particle, is then no longer the von Neumann-Schrödinger equation, but contains an additional scattering term,

$$i \frac{\partial \rho(x, x', t)}{\partial t} = \frac{1}{2m} \left(\frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2} \right) \rho - i\Lambda(x - x')^2 \rho. \quad (24)$$

If one analyzes the solutions of this equation, one finds that, for example, the Ehrenfest theorems for mean position and momentum are still valid: The motion is *not* damped, although coherence between different positions is destroyed. There is no Zeno effect.

The above equation of motion is a special case of more general equations which are studied under the topic “Quantum Brownian Motion”. These models include friction effects, a simple

	$a = 10^{-3}$ cm dust particle	$a = 10^{-5}$ cm dust particle	$a = 10^{-6}$ cm large molecule
Cosmic background radiation	10^6	10^{-6}	10^{-12}
300 K photons	10^{19}	10^{12}	10^6
Sunlight (on earth)	10^{21}	10^{17}	10^{13}
Air molecules	10^{36}	10^{32}	10^{30}
Laboratory vacuum (10^6 particles/cm ³)	10^{23}	10^{19}	10^{17}

Table 1: Localization rate Λ in $\text{cm}^{-2}\text{s}^{-1}$ for three sizes of “dust particles” and various types of scattering processes according to (22). This quantity measures how fast interference between different positions disappears for distances smaller than the wavelength of the scattered particles. For larger distances, decoherence rates are just given by the scattering rates, and thus independent of $x - x'$.

example is [10]

$$i\frac{\partial\rho(x,x',t)}{\partial t} = \left[\frac{1}{2m} \left(\frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2} \right) - i\Lambda(x-x')^2 + i\frac{\gamma}{2}(x-x') \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x} \right) \right] \rho(x,x',t) \quad (25)$$

where

$$\Lambda = m\gamma k_B T. \quad (26)$$

In typical macroscopic situations, decoherence is much more important, however. The ratio of decoherence to relaxation rate can be estimated as

$$\frac{\text{decoherence rate}}{\text{relaxation rate}} \approx mk_B T (\Delta x)^2 = \left(\frac{\Delta x}{\lambda_{th}} \right)^2, \quad (27)$$

where λ_{th} is the thermal deBroglie wavelength of the macroscopic body. This ratio has the enormous value of about 10^{40} for a macroscopic situation ($m=1$ g, $\Delta x = 1$ cm) [11].

We can conclude from these models that

- Newton’s reversible laws of motion can be derived (to a very good approximation) from strong *irreversible* decoherence.
- The appearance of classical objects has its origin in low-entropy condition in the early universe and the unique features of quantum nonlocality.
- Decoherence works *much* faster than friction in macroscopic situations.
- Although coherence is strongly suppressed, no Zeno effect (slowing down of motion) appears.

6.2 Rate equations

The exponential decay law $P(t) = \exp(-\lambda t)$ mentioned at the beginning is a special case of a general rate equation

$$\frac{d}{dt}P_\alpha = \sum_{\beta} A_{\alpha\beta}P_\beta = \sum_{\beta \neq \alpha} (A_{\alpha\beta}P_\beta - A_{\beta\alpha}P_\alpha). \quad (28)$$

Its quantum analogue, describing to dynamics of “occupation probabilities” is usually called the “Pauli equation”,

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\beta} A_{\alpha\beta}\rho_{\beta\beta}. \quad (29)$$

An obvious feature of (29) is that interference terms do not play any dynamical role. On the other hand, this cannot be true exactly, since then the von Neumann equation would lead to Zeno freezing,

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\beta} (H_{\alpha\beta}\rho_{\beta\alpha} - \rho_{\alpha\beta}H_{\beta\alpha}) \equiv 0. \quad (30)$$

To further analyze these matters let us assume that the properties α in the rate equation are macroscopic in the sense that they are *continuously observed by the environment*. The microscopic characterization is in the following assumed to be given entirely by the energy, further degeneracies are neglected for simplicity. The macroscopic feature α is measured by a “pointer” as in the two-state Zeno model above, see Fig. 9. The Hamiltonian then reads [3]

$$\begin{aligned} H = & \sum_{\alpha E} E |\alpha E\rangle \langle \alpha E| + \sum_{\alpha E \neq \alpha' E'} V_{\alpha E, \alpha' E'} |\alpha E\rangle \langle \alpha' E'| \\ & + \sum_{\alpha E} \gamma(\alpha) \hat{p} |\alpha E\rangle \langle \alpha E| \end{aligned} \quad (31)$$

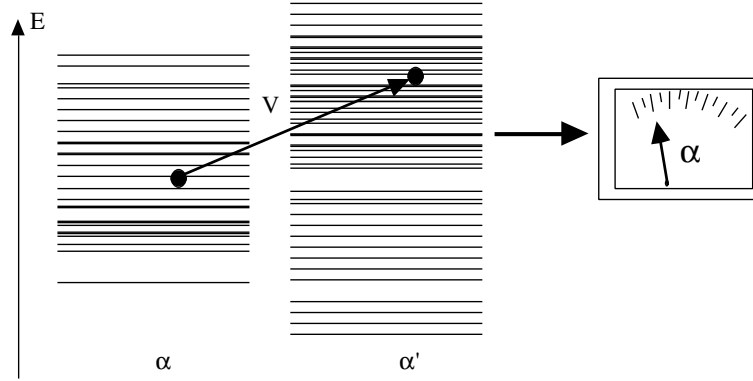


Figure 9: Transitions between groups of states are monitored by a pointer. The symbolic measurement device in the figure represents the interaction with the environment (which may or may not contain an experimental setup). Transition probabilities often follow Fermi’s Golden rule (rates governed by transition matrix elements V and level densities at resonance energy), but may be influenced by the presence of the environment monitoring certain features α of initial or final states.

Since we are interested in the limit of strong coupling to the pointer, we calculate the transition probability from property α_0 to another one, α in lowest order perturbation theory. Starting from

$$|\Psi(0)\rangle = |\alpha_0 E_0\rangle |\Phi\rangle, \quad (32)$$

where Φ is the pointer state, the transition probability is

$$P_{\alpha E} = 4 \int dp |V_{\alpha E, \alpha_0 E_0}|^2 |\Phi(p)|^2 \frac{\sin^2(E - E_0 + \gamma(\alpha)p)t/2}{(E - E_0 + \gamma(\alpha)p)^2} \quad (33)$$

(assuming $\gamma(\alpha_0) = 0$ for simplicity). This expression shows a familiar resonance factor, but now we have new resonances for each value of p with weight $|\Phi(p)|^2$, shifted from $E = E_0$ to a new value $E = E_0 - \gamma(\alpha)p$. Summing over many states with property α gives

$$P_{\alpha} \approx 2\pi t \int dE \frac{\sigma_{\alpha}(E) |V_{\alpha E, \alpha_0 E_0}|^2}{\gamma(\alpha)} \left| \Phi\left(\frac{E - E_0}{\gamma(\alpha)}\right) \right|^2 \quad (34)$$

Three limiting cases can be extracted from this expression (see also Fig. 10).

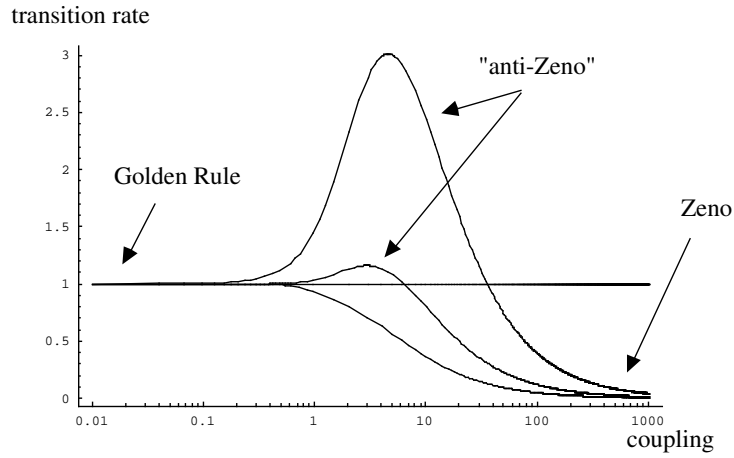


Figure 10: Continuous coupling to a pointer changes the transition rate from an initial state $|\alpha_0 E_0\rangle$ to a group of final states in various ways. For small coupling we find the standard Golden rule result (here normalized to unity). Increasing the coupling to the measuring agent may in some cases increase the transition probability by shifting the effective resonance frequency to regions with higher level density or larger transition matrix elements (anti-Zeno effect). Strong interaction always leads to decreasing transition rates (Zeno effect).

- Case 1: *Zeno limit*: For large coupling $\gamma(\alpha)$ we have

$$P_\alpha \approx \frac{2\pi t}{\gamma(\alpha)} \int dE \sigma |V|^2(E) |\Phi(0)|^2 \sim \frac{1}{\gamma(\alpha)} \quad (35)$$

Transitions are suppressed as expected.

- Case 2: *Golden Rule limit*: For small coupling, transition rates become independent of γ and the usual result is recovered,

$$P_\alpha = 2\pi t \sigma_\alpha(E_0) |V(E_0)|^2. \quad (36)$$

- Case 3: “*Anti-Zeno effect*”: If the contributions from each transition are comparable, that is, if $\sigma |V|^2 \approx \text{const.}$ in the relevant interval $[E_{\min}, E_{\max}]$ then it is easy to see that we have a smooth transition from the Zeno region to the Golden limit. If this is not the case, it can happen that in the intermediate range transition probabilities are *enhanced* above the Golden rule value. This is occasionally called “anti-Zeno effect”.

7 Summary

We have seen that unitary evolution depends decisively on interference between components of the wave function. If phase relations are lost, evolution is hindered. This leads finally to the Zeno freezing of motion. No coherence, no motion.

The destruction of phase relations can be understood as phase *de-localization* arising from *unitary* quantum evolution, if the interaction of a system with its environment is taken into account. In this way, the Zeno effect can be completely understood as a dynamical effect. No collapse of the wave function is required, but only quantum nonlocality.

Many-state systems can escape Zeno freezing. This is important for the properties of our experienced macroscopic world, but also for common “quantum” features such as radioactive decay, which happens whether or not a counter is setup to observe the decay. (In fact, in most cases Nature herself provides the necessary “counters”.)

Systems with only a few degrees of freedom are very sensitive to quantum entanglement and can therefore never escape the Zeno effect, if they are interacting with other systems. Zeno freezing can thus be used to delineate the borderline between classical and quantum objects.

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