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The Mind that Created the Bohr Atom*

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> "Philosophari volo, sed paucis, siger den nordiske natur."
> (I want to philosophize, but in a few words, says the
> Nordic temperament.) Møller, *Skrifter* (1930), 2, 364.

> "*Invention* is an *Heroic* thing, and plac'd above the reach of a low, and
> vulgar *Genius*. It requires an *active*, a bold, a nimble, a restless *mind*."
> Spratt, *History* (1667), 392.

Historians usually present their results as narrative and invoke ordinary motives, even if only a passion to solve a problem, to move the action along. Whatever physicists might think, historians still believe that particular outcomes have distinct causes. This wholesome doctrine is hard to follow, however, where the subject is creation. How can the historian or biographer hope to display a coherent narrative of a scientist's or artist's progress from one idea to another, from confusion to clarity, from knowing no more than anyone else to inventing something new to everyone? The historical actors themselves who try to fathom their course report unhelpfully that their insight or invention came suddenly, unconsciously, while they were thinking about something else.

The recent release of a portion of the correspondence between Niels Bohr and his family, especially his fiancée, during his postdoctoral stay in England in 1911/12, invites an attempt at a causal account of his path to the three-part paper of 1913 that created the quantum atom (the "trilogy"). Even with this rich documentation, however, the historian would be hard pressed to grasp Bohr's thought in transition between his initial state, when his mind, though very well stocked, contained no relevant theory or data not known to other informed theorists, to his final state, when it had generated the novel principles and methods of the quantum atom. Nevertheless, an instructive account of Bohr's creation may be given by pursuing the analogy just suggested between a quantum jump and a creative act. In place of the probabilities of the quantum description, we may put the relative significance of the various items in the creator's mind at the crucial time as estimated, qualitatively of course, by the historian. On this rough analogy, we might say that the colleagues who drew Bohr's attention to the relevance of spectra for the development of his ideas conducted an experiment that caused him to rearrange his mental furniture so as to create, and seek foundations for, the quantum atom. I'll pursue this line by first

*The following abbreviations are used: AH, Aaserud and Heilbron, *Love, literature, and the quantum atom* (2013); *CW*, Bohr, *Collected works* (1986-2008); DJH, Dansk jødisk historie; HK, Heilbron and Kuhn, Hist. stud. phys. sci. **1**, 211–90 (1979); NBA, Niels Bohr Archive, Copenhagen; PM, London, Edinburgh and Dublin philosophical magazine. Full references are given under "Works Cited" below.

sketching the furniture in place when Bohr began work on Rutherford's nuclear atom in the summer of 1912 (§1). I'll then describe the experiment and the transition or, to speak plainly, the interventions of J.W. Nicholson and H.M. Hansen (§2) before taking up the theory of the hydrogen spectrum and Bohr's revealing attempts to justify the assumptions on which he based it (§3). The essay concludes (§4) with scattered remarks about creativity.

1 A Partial Inventory of Bohr's Mind, 20 July 1912

Concerning himself

In July 1912 the high point on Bohr's agenda was his impending marriage to Margrethe Nørlund, which took place on 1 August 1912. Planning for the wedding had snagged four months earlier when Margrethe told her pious parents that her fiancé flatly refused to be married in church. The short-lived flap had the merit of causing Niels to write down some of his basic beliefs and his reasons for holding them. He had rejected religion during his early adolescence after a long battle in which he tried to bring himself to believe that salvation depended on holding fast to a few arbitrary and implausible propositions. The problem was not unusual. But Niels in his penetrating way questioned not only the doctrine but also the concept: what could it mean to have a saved soul? His doubts dissipated suddenly when he realized, with complete conviction and no room for appeal, that Christian theology was nonsense. When he told his father of his revelation, Christian, for that was the father's name, smiled. Christian Bohr, the professor of physiology at the University of Copenhagen, was an atheist, and had exposed his son to the state religion so that he would not feel himself different from other boys.[1]

But Niels was different. He interpreted his father's smile to signify not only approval, but also congratulations for correctly solving a deep and important puzzle on his own. "That smile... showed me that I too could think."[2] The experience was formative. He loved and respected his father, whose early death, just before Niels took his doctor's degree in 1911, affected him deeply. To be married in church would have desecrated his father's memory and forced him to violate his own convictions. To make certain that they would and could not be married in a religious ceremony, both Niels and Margrethe formally resigned from the Danish State Church.[3]

By raising Niels as a Lutheran, Christian had not fulfilled his marriage contract, which specified that any children he might have with his wife Ellen née Adler would be brought up as Jews. Ellen was the daughter of a rich and prominent Jewish banker, statesman, and philanthropist, David Baruch Adler, and Niels first saw the light of day in his grandparents' elegant apartment above the family bank.[4] The Adlers had passed quickly along the path of assimilation. David Adler was liberal and reformist, and, though faithful to the Jewish community, disliked its isolating practices and felt at home in the wider society. Ellen, though raised in a Jewish household, was considered to be one of the most fashionable women in Copenhagen; she was not religious and agreed to baptizing her children lest they or others think that their Jewish heritage made them lesser Danes.[5] Nevertheless, Niels could not

[1] AH, 76–80.
[2] Niels to Margrethe, 21 Dec 11, in AH, 161.
[3] AH, 73.
[4] Pais, *Bohr*, 39 (1991).
[5] Bamberger, *Viking Jews* 180 (1983); Hvidt, *Jacobsen*, 59 (2011).

have grown up without absorbing some elements of Jewish culture from his mother's family, which included her sister Hanna, a formidable educational reformer with a master's degree in physics. Niels enjoyed spending time with her. An expert on the history of Danish Jewry cites Hanna Adler's work as exemplary of the "special, very concrete idealism, expressed in an intensive and persistent striving to realize an idea or thought" that constituted the Jewish "impulse" in Danish science. In contrast with Ellen, who had little or no connection with the synagogue, Hanna had close ties to the Jewish community, which she supported financially.[6]

A more distant relative close to Niels was Edgar Rubin, a boy of his own age, who was the son of Adler cousins. Their closeness gave Edgar, who became a leading experimental psychologist, fine material to ponder; fruitfully, it appears, since Rubin's principal work, on pattern recognition and gestalt switches, turns on epistemological views reminiscent of Bohr's. Rubin broke away from nineteenth-century mechanics of the mind to assert the primacy of the mental act much as Bohr broke from classical physics to insist upon the indivisibility of the quantum of action. There are more detailed correspondences as well. Rubin insisted on a clean distinction between mental states and the external stimuli supposed to provoke them, stressed that stimuli do not uniquely determine responses, and understood that the subject's knowledge of the physical stimuli can have an influence on the object experienced, as when a gestalt switch is anticipated.[7] All three of these propositions have some similarity to Bohr's elucidations of quantum physics. They may well have been common property of the Ekliptika Circle.

The Ekliptika was a club of 12 overachieving university students led by Rubin and devoted to discussions centered on teachings of their philosophy professor, Harald Høffding, who raised big questions in an introductory course that for many students marked the transition from school learning to university thinking.[8] At least half of the figures in this zodiac were of Jewish descent. Besides the Bohr brothers and Rubin, there were Rubin's first cousins, Lis Rubin and Einar Cohn, and also Vilhelm Sloman, all of whom except Cohn made distinguished academic careers.[9] (Cohn, a mathematical economist, succeeded his uncle, Marcus Rubin, as head of the Danish Statistical Bureau.) The subjects pursued by members of the Ekliptika ran from mathematics (Harald Bohr) through physics and experimental psychology (Niels Bohr, Rubin), to linguistics and art history (Lis Jacobsen, Sloman). Jacobsen née Rubin, who received her Ph.D. in 1910 when already married, became an authority on the Danish language and Nordic runes. Sloman became director of Kunstindustriemuseet (Museum of Applied Arts), whose collections owe much to Jewish philanthropy.[10]

The remarkable representation of students of Jewish descent in the Eklptika and their subsequent high academic achievement were consequences of the traditional Jewish emphasis on study, the tendency of liberal reform Jews to assimilate, and the relative tolerance of Danish society. A cartoon of 1910 perfectly represented the balance required for success. It showed the Finance Minister, Edvard Brandes, who was Jewish and boasted a Ph.D., in conversation with the indolent Foreign

[6]Rerup, in *Indenfor murene*, 215 (1984); on Hanna Adler, Pais, *Bohr*, 38 (1991), AH, 20–4, and Margrethe Bohr, *Interview* (1963), session 1.

[7]Rubin, *Synsoplevede figurer* (1915); *Experimenta* (1949), 9-11, 12–13 (quotes, text of 1927), 14–15, 79–81. Cf. Witt-Hansen, Dan. yearb. phil. **17**, 45 (1980).

[8]Witt-Hansen, Dan. yearb. phil. **17**, 42 (1980); Rindom, *Høffding*, 65–6 (1913).

[9]Hvidt, *Jacobsen*, 55–6 (2011).

[10]Christensen, Rambam **20**, 59–73 (2011).

Minister, Count Raben-Levetzau. "Jews have almost the same chance here in [Denmark] as, say, counts," says the count." "Not entirely," replies the doctor, "stupid Jews have absolutely no chance."[11] They had to try harder. Of course, the members of the Ekliptika were not chosen at random from among the best students at the University. Five of the members were cousins of one sort or another, in keeping with the tendency, even among assimilated Jews, to stick together. At least two of the gentile members were drawn into this large family, the brilliant brothers Niels Erik and Poul Nørlund, country boys raised by pious parents, who distinguished themselves in mathematics and history, respectively. Their younger sister Margrethe first met Niels at the home of Edgar Rubin; and it is very likely that their subsequent engagement was fostered by the calculations of Harald and Niels Erik.[12]

"The most truly faithful friend of the whole family," so Harald wrote Niels, was also of Jewish descent.[13] He was Valdemar Henriques, a former student of Christian Bohr and, in 1911, his successor as physiology professor at the university. With help from Hanna Adler, Henriques was to play a part, perhaps a major one, in persuading the government to set up a chair for theoretical physics for Bohr. And, as chairman of the board of the Carlsberg Foundation in the 1920s, he would help to provide the money to equip Bohr's Institute for Theoretical Physics – the land for which was acquired largely with the help of Jewish philanthropists.[14]

Despite the general acceptance of "Viking Jews" – members of well-to-do families long settled and largely assimilated in Denmark – the "Jewish Question" reopened during Bohr's student days. It had been dormant for two generations following the adoption of a new constitution in 1849 that relieved Jews of all civil disabilities. Among its consequences was the return of David Adler to Copenhagen from London, where he had immigrated to pursue his business interests free from the restrictions he had experienced in Denmark. He brought with him an English wife, Jenny Raphael, from a German-Jewish banking family established in England.

The Jewish question reopened largely because of the influx of Polish and Russian Jews driven West by pogroms. Although their numbers in Denmark were small, they amounted by 1911 to a third of its Jewish population; a rise from essentially zero in 1901 to 1600 in ten years (the Vikings then numbering 3200) and to parity, at 2700 each, in 1921. In contrast with the established Jews they were ignorant, poor, socialist, Zionist, and orthodox, spoke Yiddish and acted East-European, all unwelcome traits to most Danes, Jewish or Gentile.[15] Still, they were persecuted brethren, and even so acculturated and anti-Semitic a Jew as Georg Brandes, the leading Danish man of letters of his time, felt obliged to help them.[16] The Russian immigration thus forced the lurking problem of reform Judaism, assimilation, to the fore.

The problem was being aired in an improbably popular play, *Indenfor murene* ("Inside the walls"), by the Zionist Henri Nathansen, when Bohr returned home during Easter vacation of 1912.[17] He could not have been ignorant of this play, which had become the talk of the town, although, since tickets were almost unprocurable,

[11] Quoted from *Klods-Hans* by Rerup, in *Indenfor murene* (1984), 188–9.
[12] AH, 9, 57.
[13] Letter of 30 Jan 12, AH, 69, and ibid., 52–3.
[14] Rerup, in *Indenfor murene* (1984), 196-7; AH, 94.
[15] Jacobsen, Rambam **16**, 13–17 (2007).
[16] Knudsen, Rambam **7**, 6–8, 13–15 (1998).
[17] Arnheim, DJH **30**, 42–7 (1990); Lassen, DJH **25**, 21–4 (1987).

he may not have seen it immediately.[18] But as it ran for over 500 nights, he would have been able to see it after his definitive return to Copenhagen in the autumn of 2012. The Jewish heroine of the play, Esther, falls in love with a gentile professor whose lectures she attended, just as Ellen Adler had done with Christian Bohr. Esther's engagement distresses both families, but the action takes place primarily "within the walls," in the warm, cozy, middle-class Jewish home that Esther's desertion threatens to destroy. Similarly, the story of Niels and Margrethe unfolded within the Adler side of the family; she lived with Ellen for some time before her marriage and saw very little of the Bohr side of the family, perhaps because it included a number of Lutheran theologians.[19] But whereas Margrethe joined the assimilated Adlers easily, Esther could not be comfortable with her bigoted in-laws. Nathansen ends his play ambiguously. Esther's mother prays, "Dear God, let my child live happily," but receives no hint that God will respond favorably.[20]

Esther's interfaith marriage was not atypical of Viking Jews in the decades around 1900 – in almost 40 percent of weddings involving a Jew during the period the other party, usually the male, was a gentile.[21] Though the union of a Jewish woman with a gentile academic was rare, Bohr knew at least two such cases, his parents and the Jacobsens. Lis Jacobsen's story is particularly cogent. Her grandparents kept kosher; her father Marcus rejected orthodoxy but remained in the Jewish community, married an observant Jewess, and celebrated the holidays. Though close to the arch-assimilationist Brandes, he would not allow Lis to attend Christian religious instruction in school.[22] Curious to know what her playmates were learning in these prohibited classes, she asked her father for permission to participate. A puzzled Marcus Rubin turned to the universal sage, Høffding, who suggested that he give Lis some Christian books to read. That satisfied her. She grew up an unbeliever and liberal do-gooder, and aimed to be a schoolteacher until she fell in love with a gentile intellectual from a pious family. What would his mother say to his engagement to a Jewish woman with no faith? His answer showed that like Brandes, with whom she carried on a lengthy correspondence, she would not be able to shake off her Jewish identity. "[My mother] will treat you as a holy woman from the Old Testament."[23] Later Lis Jacobsen promoted Jewish causes and is now cited along with Hanna Adler among the most distinguished Jewish women in Danish history.[24]

Nathansen's observations convinced him that Jews had developed racial traits that enabled them to survive their persecution. He admired the survival traits, despised the cowardice or conformity that had caused many to assimilate, and welcomed the Zionist movement as a means of preserving a valuable way of life. The leading Jewish traits as itemized in Nathansen's sensitive biography of his friend Brandes are strength and joy in work. To the world Jews are fierce competitors, who can come across as over-critical, domineering, and arrogant. "In every Jewish boy there is a little Napoleon." The world also knows Jews as champions of equality, truth, justice, freedom, and human rights.[25] With family and friends, "in-

[18] Riis, Rambam **16**, 34 (2007); Bille, DJH **21**, 15 (1986).
[19] M. Bohr, *Interview*, session 1.
[20] Nathensen, *Indenfor murene*, 125 (1965).
[21] Blüdnikow and Jørgensen, in Jørgensen, *Indenfor murene*, 134–6 (1984); Bamberger, *Viking Jews*, 98, 100, 105 (1983).
[22] Thomsen, Rambam **11**, 33, 38 (2002); Hvidt, *Jacobsen*, 22–3 (2011).
[23] Hvidt, *Jacobsen*, 24–5, 37–9, 44–7, 70–1, 73 (quote) (2011).
[24] Sandvad, Rambam **10**, 78–9 (2001); Rerup, in *Indenfor murene*, 213–14 (1984); Hvidt, *Jacobsen*, 31 (2011).
[25] Nathansen, *Jude*, 39–41, 46 (second quote), 78, 88, 94 (1931).

side the walls," Nathansen's Jews have an "intimate special life... whose passwords [are] 'respect' and 'discipline' – respect for tradition, discipline in the family." There competitiveness turns to humor, irony, satire, word play, banter, "wily, equivocal, ambiguous, double-edge wit combined with irony and self-irony." "The world of the mind was the home of the homeless Jewish people, the life of the mind their only free state....From the special exclusivity of this life of the mind Jewish 'chutzpa,' boldness, something between courage and insolence, has developed, and also Jewish 'chain,' the artistic, sensitive union of grace and taste, something between enchantment and enticement."[26]

Although Brandes denied that Jews were a race with special traits, nonetheless in his biographies of influential Jews, like Ferdinand Lassalle and Benjamin Disraeli, he presented a list similar to Nathansen's, and in 1912, the year of Nathansen's theatrical success, gave lectures at the University of Copenhagen on the Jewish spirit in Denmark. The list included talkativeness, adaptability, passion, rationality, dry wit, boldness, and impudence. Brandes was bold and impudent enough to add Christianity to the list; for what is Christianity, he asked, but "ancient Jewish culture, ancient Jewish barbarism"?[27] He did not think himself Jewish. Once when among friends, excited by their admiring attention, he broke out, gesturing with his palms upward, "would anyone one of you take me for a Jew who did not happen to know that I am one?" To which Nathansen replied, neatly adapting Isaac's puzzlement in Genesis 27:22, "would anyone who saw G.B. in this psychological moment doubt that the voice is Jacob's and the hands Esau's?"[28] Like Esau, Brandes was selling his birthright; like Jacob, he was seeking it; like Isaac, he was blind to the situation.

Bohr's visit to Copenhagen at Easter 1912, after only a few weeks at Rutherford's laboratory in Manchester, made him face up to a religious problem of the same intensity as the difficulty that had precipitated his loss of faith. Margrethe's pious mother was very much upset by her daughter's decision not to be married in or by the church. After his return to Britain, coincidentally while visiting his Raphael relatives in Edinburgh, Bohr had to write to his future mother-in-law explaining why he repudiated her religion.[29] The taste of the unpleasantness occasioned by her beliefs stayed with him. For a time he wanted to write a book on religion, to warn people that it was not true and that they should not build their lives upon it.[30] Thus he was particularly aware of the necessity and difficulty of reconciling conflicting religious beliefs and cultures just before he encountered the contradictions of the nuclear atom.

Like Lis Jacobsen, Margrethe Nørlund had planned to be a schoolteacher. Unlike Jacobsen's fiancé, however, Margrethe's did not encourage her to go to the university; her formal education ceased at the academy where she, and Jacobsen before her, studied pedagogy.[31] This educational shortfall made her fear that she might not be able to play her part in marriage to a compulsive intellectual like Niels. When she confessed this doubt to Ellen Bohr, she received the reassurance that Niels did not require a well-educated wife but a woman who loved him and respected intellectual work. Ellen highlighted this lesson by reference to half a dozen saccharine heroines

[26] Quotes from, ibid., 42, 50, 60, resp.
[27] Gibbons, in Hertel and Kristensen, *Activist* (1980), 61, 72, 94; Dahl and Mott, ibid., 325 (Brandes' lecture).
[28] Nathansen, *Jude* (1931), 103–4; cf. Knudsen, Rambam **7**, 8, 16 (1998).
[29] AH, 72–3.
[30] M. Bohr, *Interview* (1963), session 1.
[31] Hvidt, *Jacobsen*, 36 (2011); AH, 10.

from Dickens novels – Little Dorritt, Agnes from *David Copperfield*, Florence from *Dombey and Son*, and so on – undereducated, perhaps, but informed enough to sacrifice themselves for the good of others.[32]

Niels created a similar Margrethe from a richer set of literary exemplars. He sent her a copy of *David Copperfield* in English, to improve her command of the language, perhaps, and to call attention to the magnificent patience of Agnes, who waits while David marries a pretty empty-headed improvident creature whose incapacity to be anybody's wife is obvious to everyone but David.[33] Fortunately she is also sickly and her death clears the way for and to Agnes. How patient was Margrethe? Niels asks whether, if they were to go to Iceland together and he had to leave, she would wait for him. She replied to this odd question in the literary idiom in which they clothed their emotions. "I will come to you, Niels, as Solveig came to Peer Gynt." This was to grant more than was required, however, since the innocent Solveig, who offered herself freely to the vagabond Peer, wasted her life waiting for his return. The catechism continued. "Will you care for my work?" Margrethe had not been to the university but she had read Carlyle's *On heroes and hero worship* and knew how to treat great men. She answered, "Dear Niels, I cannot at all describe to you how much I love you and how much I love your work." But will you be a mother to my students?[34] "I set no limit at all to how much I wish that I could be allowed to be a mother to your students." Will you pay my debts, "all the debts that my poor soul might incur?"[35]

Although Niels repeated this question several ways, he never specified the obligations he needed Margrethe's help to discharge. I interpret them to be the repayment, by success, of the belief in his abilities entertained by his family, teachers, and friends. To his mother he was a "rare treasure," to his father "gold," to his brother, "the greatest and wisest human being we have known."[36] The family not only admired his abilities but also helped him develop them. He dictated most of his thesis to his mother. His father put his laboratory and mechanic at Niels' disposal when Niels competed for a prize offered by the Royal Danish Society of Science. (He won the gold medal, which caused Christian to quip that while he, having won a lesser prize, was silver, Niels was gold.) The entire family helped in computing tables, doing calculations, and writing out fair copies. Margrethe became part of this machinery even before her marriage, as witnessed by a set of corrections of Niels' doctoral thesis on the electron theory of metals in her hand.[37] Harald helped with calculations for the thesis as did Margrethe's brother; and once again, as in the gold-medal paper, Bohr's little group produced a capital piece of work. Consequently, when he arrived in Cambridge in the fall of 1911 to continue with the electron theory under J.J. Thomson, he felt confident that he would be able to distinguish himself enough to justify the belief in his capacities that had sustained him in Denmark.

From this point of view, his two terms at Cambridge were not successful. Thomson, though cordial, had moved away from the theory of metals and, in any case, did not have the patience to listen to a long-winded foreigner explain in shaky English why his old theory could not stand. Instead, Thomson set Bohr a pointless little

[32] AH, 134.
[33] AH, 107.
[34] AH, 157.
[35] AH, 160.
[36] AH, 134, 127, 12.
[37] AH, 136, 155–6.

experimental investigation to acquaint him with the working of the Cavendish Laboratory, which did not work for him, as he did not know the English names for the tools and apparatus he needed and no one seemed disposed to help. Worst of all, his several attempts to publish his thesis in English failed; and as neither Thomson nor the other mathematical physicists in Cambridge, Joseph Larmor and James Jeans, consented to read it in the rough English translation he had brought with him, none of them was able to take his measure. To the end of his life he remembered the snub, which, as he recognized, arose from the incongruity of a mature visiting postdoc in the Cambridge system. Thomson arranged for his admission to Trinity College but when he dined there, which he seldom did, he had to sit with younger men with whom he had very little in common.[38]

Bohr's translation to Manchester was therefore a new beginning, a new chance to prove his mettle on the international stage and to begin to discharge his imagined debts. Margrethe's part in the pay-back would be to support and reassure him, to smooth out his mood swings, and to act as sounding board and amanuensis. Fortunately for their peace of mind, two months before their marriage Bohr gave up the theory of metals and the exercises in radioactivity prescribed for neophytes in the laboratory. Instead he devoted himself to transforming the nuclear model of the atom, which Rutherford had revived in 1911 to interpret the scattering of alpha particles, into a competent atomic theory. At the time, the leading atomic theory, which Thomson had been developing for a decade, offered more or less plausible, qualitative accounts of radioactivity, chemical periodization, and the formation of simple molecules, and successful quantitative computations of the passage of X rays and beta rays through matter.[39] Thomson again made the competition Bohr had to meet.

Preliminary engagement with transferring Thomson's techniques to the nuclear atom precipitated a rapid alternation of psychological ups and downs. When down, Bohr condemned his ideas as "only indications of the most fleeting fantasy." But then this fantasy, this creative imagination, was, he told Margrethe, "the most valuable and only thing that I possess, and with that I come to you, and ask you... to redeem it for me, for us. My own, my little darling, tell me, will you understand that it is at the same time [a matter of] the deepest seriousness and the greatest happiness in life for your Niels?" Within a day or two he was up again. He had written "a first little superficial draft" for discussion with Rutherford. He would bring it home, they would work on it together, and "try to put some of our happiness into it."[40] It was the first draft of the quantum atom.

Concerning Physics

The physics in Bohr's mind in July 1912 consisted primarily of the results of his doctoral thesis and of the wide reading in electrodynamics and radioactivity he had done in Cambridge and Manchester. Also, almost certainly, he had news of the deliberations of the Solvay Council, which had met in October 1911 to discuss problems of "radiation and the theory of quanta." Although this meeting, a small private gathering that assembled at the invitation of a Belgian chemical tycoon, had not yet published its proceedings, Bohr knew about them from Rutherford, who

[38] AH, 135–41.
[39] Heilbron, Arch. hist. exact sci. **4**, 269–80 (1968).
[40] Niels to Margrethe, 16 and 19 Jul 12, in AH, 92–3, 166–7.

served on the scientific committee of the Institut International de Physique Solvay founded in 1912, and also, perhaps, from James Jeans and/or Martin Knudsen, an experimentalist at the University of Copenhagen, both of whom attended the Council.

From his thesis Bohr took the conviction that Newtonian mechanics and Maxwell's electrodynamics did not furnish concepts adequate for a description of the microworld revealed by the experimental detection of the electron. He identified the principle of the equipartition of energy, which seemed an inescapable consequence of the most general theories of "classical physics" (as Max Planck and other Solvay participants were beginning to call their elegant but inadequate heritage), as the prime locus of failure. For not only did it give a wildly wrong answer to the problem of heat radiation that had driven Planck to invent his quantum theory, it also made the magnetic properties of metals inexplicable, indeed, impossible. Bohr liked this demonstration of impotence (it gave rise to the only diagram in his thesis) because he had discovered it himself.

Contrary to Paul Langevin's account of para- and diamagnetism, which, on its publication in 1905, was heralded as a major conquest of received physics, Bohr showed that a rigorous application of the equipartition of energy wiped out all effects of external magnetic fields on electrons in metals whether bound or free. That did not bother him. As a student of Høffding's, he was perfectly prepared for the eventual fatal failure of every theory, however powerful. Høffding had taught the Ekliptika that the concept of a secure fact, and the notion of a complete theory, are ideals, even myths. Sooner or later, a promising line of reasoning will hit an immovable contradiction or impediment. "Neither [a secure fact nor a complete theory] is given in experience, nor can either be adequately supplied by our reason; so that, above and below, thought fails to continue, and terminates against an 'irrational.'"[41]

The sunnier mathematical physicists of the Cambridge school were not prepared to admit the incurable failure of their hard-won methods. That does not mean that they regarded the theories produced by their methods as faithful transcriptions of the operations of nature. Rather, they believed that they had tools and concepts that enabled a theorist with sufficient ingenuity to give a quantitative account of any physical phenomenon. And so they neutralized equipartition without disturbing the physical principles that anchored it. Equipartition was inevitable, they granted, but not imminent; it might take millions of years, perhaps longer than the life of the universe, to set in. Thomson and Lord Rayleigh entertained this awkward concept and Jeans, who developed it furthest, had the unenviable task of reporting on it to the Solvay Council immediately after Lorentz had argued the need for a concept like Planck's quantum to avoid the disastrous consequences of equality among electrons. One of the arguments brought against delayed equipartition by the Solvay participants was the difficulty in discriminating cases in which it sets up immediately (where classical physics worked) from those in which it may not do so for millennia. As Poincaré put it to Jeans, he would need an elaborate collection of parameters to define the pace of equilibrium in the various cases, whereas, in proper methodology, the physicist should aim to do with as few parameters as possible.[42]

Poincaré makes a good barometer of the quantum climate in 1912. He was not close enough to the problems of radiation to have been among the invitees suggested

[41]William James' epitome of Høffding's epistemology, in Høffding, *Problems* (1905), xi.
[42]Jeans, in *Théorie* (1912), 62–71; Poincaré, ibid., 77; Nernst, re Rayleigh, ibid., 51.

by Walter Nernst, who had secured Solvay's support, or by Planck; but owing to his grasp of physics and his original ideas about scientific method, he was an excellent choice when Solvay deemed that the French contingent needed strengthening.[43] Poincaré returned to Paris from Brussels unconvinced of the necessity for discontinuities required by Planck's enabling hypothesis ($E = h\nu$, energy is proportional to frequency) and worried that it limited the reach of the every-day tool of the mathematical physicist, the differential equation. In Einstein's and Lorentz's version of Planck's enabling hypothesis, the energy of an atomic harmonic oscillator (an electron on a perfect spring) is proportional to its vibration frequency. In changing its state, such an oscillator could not move continuously, and so eluded description by differential equations as assumed in the classical theories, but would be liable to discontinuous jumps. Jeans' theory was no way to avoid the difficulty: "as it predicts nothing, it is not controlled by experiment, but it leaves unexplored all the laws that it is designed not to contradict."[44]

The Solvay discussions disquieted the logician in Poincaré. He observed that his colleagues used both the old and the new physics to support the same theory. "[D]o not forget [he admonished them] that there is no proposition that cannot be easily demonstrated if you introduce two contradictory principles into the demonstration."[45] Back in Paris and able to disentangle the mess at leisure, he located the primary difficulty in carrying quantum theory forward in the exchange of energy between oscillators of different frequencies, say e and f. How does a quantum he become a quantum hf? The change requires an intermediary capable of giving and taking energy continuously, for example, molecules able to collide inelastically with the oscillators. To achieve equilibrium in this way would seem to require a pre-established harmony, or the eons that Jeans invoked to avoid quanta altogether. The transfer of quanta from an atom to the ether implied a harder puzzle: since radiation emitted at different times lacks the coherence necessary to give rise to interference phenomena, we must assume that an individual quantum can interact with itself. Quite apart, therefore, from the "laziness of our minds, which dislike changing their ways," the quantum theory discussed by the Solvay Council had very serious problems. Would its partisans manage to save it? Will an entirely different explanation be found? "Will discontinuity reign over the physical universe... or will it turn out to be only apparent...? To try to give an opinion on these questions now would be a waste of ink."[46]

Bohr might have looked up these opinions of Poincaré, which had been published and noticed in several prominent places by July 1912, for he knew Poincaré's work, particularly on thermodynamics, which he had subjected to a careful reading when he was in Cambridge. "It is so amusing," he wrote Margrethe, although he did not agree with it all. "Poincaré is a very great man, but scarcely one of the very greatest; but perhaps his mathematical genius comes through most clearly precisely in the difference there seems to be between the ease with which he treats the logical side and the way he treats what I would call the real side of what he writes about." Poincaré's logical puzzles about energy exchange and self-interfering quanta would not bother Bohr as he designed the quantum atom.[47] Nor would the great variety

[43]Heilbron, in Lambert, *Workshop* (2014).
[44]Poincaré, *Dernières pensées* (1924), 166, 174 (quote), 185; cf. Gray, Poincaré, 150–2 (2013).
[45]Poincaré, in *Théorie*, 451 (1912).
[46]Poincaré, *Dernières pensées*, 179–80, 192 (quotes) (1924).
[47]Niels to Margrethe, 12 and 17 Dec 11, in AH, 42–3.

among the formulations of the quantum hypothesis entertained by the few Solvay participants who accepted that discontinuity reigned over the microworld world. The greater the variety of formulation, the wider the license of application. Planck himself had given his enabling hypothesis in two forms: in the earlier, the oscillators emit and absorb discontinuously; in the later, only emission is so afflicted. Also, Planck shifted the category of quantization from energy to action and left open the question whether emission could occur in many quanta or only one at a time. Nor was it clear that the quantum, whether of action or of energy, was measured by Planck's h. Arnold Sommerfeld gave reasons to prefer $h/4$ or $h/2\pi$. But then, as Poincaré remarked, "the only connection between [their theories] is that both use the letter h." He allowed himself further sport with Sommerfeld's formulation of the photo-effect, which made the time over which a fast electron loses energy in a collision shorter the greater its velocity. "If this law were applicable to railway carriages the problem of braking would take an entirely new form."[48]

In a word, "quantum theory" in 1912 was a hodge-podge of postulates among which atom modelers could choose what best suited their needs. Bohr unexpectedly found himself in this position in Manchester when he set aside the theory of metals to improve a calculation of the loss of energy by alpha particles passing through matter that another senior research man there, Charles Galton Darwin, had in hand. The problem, in which point alpha particles collide with nuclear atoms, was peculiar to Rutherford's laboratory: nowhere else did anyone take his model seriously enough to invest energy in working out its implications in detail. Darwin supposed that for his purposes he could consider bound electrons as free. Bohr objected that the binding seriously affected the exchange in energy when the collision time – the duration of the alpha's passage by the target atom – approximated the period of the perturbed vibration of the electron around its equilibrium orbit. He had this insight from his considerations of the interactions of bound and free electrons in metals, and expressed the phenomenon, with his characteristic gift for vague deep analysis, as similar to the anomalous dispersion of light. The passing particle's force on the electron thus resembled the oscillating electric field of a light wave. In missing this analogy, Darwin also lost the possibility of learning something about the binding of atomic electrons from the measurements he was trying to explain.[49]

Thinking he could derive a useful relation between known frequencies of anomalous dispersion and atomic parameters, Bohr tried to calculate the response of electrons circulating around a nucleus to a changing electric field. He seems to have expected an easy victory and a quick return to the electron theory.[50] He soon discovered that the model did not permit the calculation: the perturbed vibrations of the electrons around their equilibrium orbit that occur in its plane are unstable mechanically. If a ring contains two or more electrons, its perturbed vibrations contain modes that grow without bounds and tear the atom apart. This catastrophe has nothing to do with radiative instability. If the particles circulating around Saturn repelled one another, its rings would not be stable. The fact, though not the timing, of Bohr's discovery of the radical mechanical instability of the nuclear atom appears in the heading of a file of computations, "Temporarily abandoned, since the computation breaks down over the system's instability, [and] cannot be continued without

[48] *Théorie*, 377, 381 (1912); AH 151; Poincaré, *Dernières pensées*, 190 (1924).
[49] HK, 237–41.
[50] HK, 237–8.

some other hypothesis."[51]

Here was another case in which "thought fails to continue, and terminates against an irrational.'" The discovery, which would have made most physicists choose another topic, delighted Bohr. Failure pointed the way: "it could be that perhaps I've found out a little about the structure of atoms."[52] He had already recognized that the nuclear atom allowed a clean distinction between ordinary phenomena, which involved the electronic structure, and radioactive phenomena, which had their seat in the nucleus. With the information, imparted to him by the physical chemist Georg von Hevesy, who was also in Manchester during the creative summer of 1912, that some substances distinctly different in radioactive properties and in inferred atomic weight were nonetheless chemically inseparable, Bohr worked out for himself the concepts of isotope and atomic number (N).[53] The identification of N with position in the periodic table was eased by the conclusion, which Thomson had set up from calculations based on his model atom and Rutherford had clinched in the assumptions of his scattering theory, that the number n of electrons in a neutral atom of weight A is about equal to $A/2$. On average throughout the periodic table ΔA, the increase in weight from one element to the next, is 2. Thus $\Delta n \approx 1$ and since, on Rutherford's model, the nuclear charge Ze must be equal to the total electronic charge ne, $\Delta Z \approx 1$. But by definition $\Delta N = 1$. The nuclear model takes $Z = N$ and represents atomic number by the charge on the nucleus.

Before its arrest, the continuous line of thought stopped by the mechanical instability of the nuclear atom had produced Thomson's model and its semi-quantitative account of the periodic table, Rutherford's exact scattering theory of alpha particles, and the powerful concepts of isotope and atomic number. These concepts and the impasse to which stability calculations led seemed promising enough to Bohr to cause him to shelve the electron theory of metals and to adopt a discontinuity or irrationality as a basis for a new line of continuous advance. He disclosed this basis to Rutherford in a precious document, the "Rutherford Memorandum," written shortly before he left Manchester for marriage on 25 July. To motivate his discontinuity, Bohr explained that the nuclear atom lacked not only mechanical stability but also the means to fix its size. A constant (K in Bohr's notation) with the dimensions of action could be combined with the parameters defining the electron (its charge e and mass m) and the nucleus (Ze) to obtain firm values for the radii of the electron rings. Since K characterized the interaction of electrons and their relations with nuclei, it might also figure in a rule that, when satisfied, preserved electron rings from destructive vibrations and released them from their obligation to radiate. Bohr looked to the quantum hodge-podge for a suitable rule. He found it in a strained analogy to Planck's restriction on the oscillators at the heart of his radiation theory: in their "permanent" or ground state, achieved after they have radiated away all the energy that nature allows them to dispose of, every electron bound in a nuclear atom, regardless of the radius of its ring, has a kinetic energy T equal to K times its orbital frequency ω.[54] In this formulation, as in his deduction of atomic number, Bohr imagined atoms to be built up by the successive capture of electrons by an initially bare nucleus, in the manner that alpha particles become helium atoms.

Having laid down the K-condition, Bohr supposed that ordinary mechanics de-

[51] HK, 242.

[52] Niels to Harald, 19 June 12, in HK, 238, and *CW* **2**, 559.

[53] Cf. Hevesy, Nature **131** (7 Jan 33), 4; Hevesy to Bohr, 15 Jan 13, in *CW* **2**, 528.

[54] "Rutherford Memorandum," in *CW* **2**, 136–58, on 147.

termined the shape and frequency of the electron orbits. Not knowing the value of K, however, he could not progress very far quantitatively. The line of least resistance followed Thomson's concept of a non-polar molecular bond as a sharing of electrons between atoms. Bohr pictured the bond as a ring of easily detached electrons centered on and perpendicular to an axis defined by the nuclei and the tightly bound electrons. Using ordinary mechanics augmented and restricted by the K-condition, Bohr computed the binding energies of the electrons in hydrogen and helium atoms and molecules. The calculations showed that hydrogen electrons would lose, and helium electrons gain, energy in going from atoms to molecules, wherefore H_2 exists and He_2 does not. In this calculation Bohr did not need to know the value of K, which occurs only as a common multiplier. Apparently he tried to estimate K by comparing his calculation of the loss of energy in making a mole of molecular hydrogen from its atoms with measurements of the mole's heat of formation. Working backwards from the numbers he gives he would have found that $K \approx 0.6h$. This is also the value he would have found by equating the frequency of a prominent resonance line in the spectrum of H_2 with the orbital frequency of its binding electrons.[55] This was not, however, the value he would need.

Finally, and most significantly, Bohr recommended his K-condition to Rutherford because it allowed a more progressive approach to the periodic properties of the elements than Thomson's. Thomson explained periodicity as a consequence of similarity of structure of inner electron rings; elements of the same family do not show the same face to the world. Thomson arrived at this conclusion because on his model the stability of outermost rings improves, up to a point, by increasing the number of electrons in inner rings. But since for Bohr chemical properties followed primarily from the structure of the outermost ring, he took as confirmation of his K-condition that, in combination with the ordinary mechanics with which it conflicted, it required additional electrons to go *outside* a completed ring. And he was jubilant to discover that energy considerations limited the innermost ring to seven electrons, which, with some good will, might be interpreted as eight, a figure prominent in the periodic arrangement. "[T]his seems to be a very strong indication of a plausible explanation of the chemical properties of the elements. . . .The difference in this respect between the atom-model considered [Rutherford's] and J.J. Thomson's atom-model is very striking, and seems to make it impossible to give a satisfactory explanation of the periodic law from the last mentioned atomic-model."

The argument might strike the reader more strongly than it did Bohr since his argument limiting the population of the innermost ring is plainly wrong. It claims that the total energy per electron changes from negative (at $n = 7$) to positive (at $n = 8$), whereas at the end of the Rutherford Memorandum Bohr proves the elementary theorem that the total energy of an electron in a circular orbit in a nuclear atom is always equal to the negative of the electron's kinetic energy, and so can never be positive.[56] The error no doubt was a product of haste and eagerness, a desire to seize a decisive result from his new line of work to bring back to Copenhagen as a nest egg.

[55] HK, 248–52.
[56] HK, 245–6.

Concerning culture

To collaborate with his creative imagination in integrating these many fragments of physics, Bohr had in stock a well-developed concept of scientific truth. He thought that he could prove logically that there must be aspects of our experience that will elude rational explanation forever, "that there must be something a human being does not understand." This demonstration pleased him greatly; as he wrote to Margrethe's mother in attempting an explanation of his rejection of religion, "[life] would be so infinitely trivial if I thought I could understand it."[57] The doctrine that truth lies in a deep well whose bottom we will never reach, and that, consequently, science must content itself with seeking laws rather than true causes, was commonplace around 1900, indeed, probably the opinion of a majority of physicists. Bohr would have found it expounded magisterially in the translation of Poincaré's *Science et hypothèse* published in 1905 with a preface by Joseph Larmor, and, less wittily, in Larmor's *Aether and matter*, which Bohr admired for its breadth of view and, contrarily, for its author's "very great gifts for making things difficult."[58] But their views, varieties of what has been labeled "descriptionism," had no deeper ground than the abundant evidence of the limited capacity of the human mind, and did not provide a reasoned definition of what, under the circumstances, should be taken as "truth." "At the present [1906], we have no idea of what the word may mean."[59]

Bohr's concern with epistemological questions developed during his undergraduate studies with Høffding. It appears that he formed the precocious resolve to write a book on the nature of knowledge and so may well have been the "young friend" of whom Høffding told a fellow philosopher he expected "so much in a philosophical way." That was in 1902, when Niels was 17. This young friend had not scrupled to criticize Høffding's big book on religion just as (if he were Niels) he would not hesitate to correct an error in Høffding's textbook on logic.[60] Bohr visited the professor while still a student, perhaps to discuss logic or some other problem in philosophy or life, for Høffding felt the responsibility of acting in loco parentis and invited all his charges to write him about any problem that bothered them. Bohr had privileged access to him since much of Høffding's intellectual and social life centered on the fortnightly discussions of a quartet of professors composed of himself, Christian Bohr, the physicist Christian Christiansen, and the philologist Vilhelm Thomsen. Niels and his brother Harald listened to their wide-ranging conversation, which gave them the precious experience of observing experienced scholars failing to reach certainty about the great issues of science and philosophy.[61] The first of these issues concerned "the nature, condition and limits of knowledge, the nature and worth of evidence, and the principles which underlie our valuation of human actions and institutions," that is, the problem of Truth.[62]

Bohr took the formal problem of truth from Høffding, who in turn began with Søren Kierkegaard. Bohr rated Kierkegaard's *Stages on life's way*, which Høffding took to be representative of Kierkegaard's philosophy, as one of the best books

[57] Bohr to Sophie Nørlund, 1 May 12, both quotes, in AH, 77.
[58] Bohr, *Interview*, 21, 27 (1962): "I loved that [making things difficult] in some ways because it is a way to think over things" (21).
[59] W. James, *Pragmatism*, 74 (1907); for descriptionism, Heilbron, in Bernhard et al., eds, *Science*, 52–7 (1982).
[60] Høffding to Tönnies, 27 May 02, in Bickel and Fechner, *Briefwechsel*, 90 (1989); Aage Petersen, cited in Witt-Hansen, Dan. yearb. phil. **17**, 48–9 (1980) (epistemology), 49–51 (logic).
[61] Bohr, *CW* **10**, 309, 319 (texts of 1928, 1932), and Høffding to Bohr, 22 Nov 06, ibid., 505; Rindom, *Høffding*, 84 (1913), and *Samtaler*, 56–7 (1918).
[62] Høffding, Int. jl ethics **12**:2, 137 (1902).

ever written. Høffding and Brandes, who otherwise had little but mutual respect in common, also admired it, and defined and overcame religious crises in early manhood with its help.[63] Further to Bohr's pedigree in Danish philosophy, he loved a philosophical romance by Kierkegaard's major patron, Poul Møller, a professor of philosophy considered by many to be the archetypical Danish writer of his time, "a humanist whose instrument was the binocular of poetry and thought."[64]

Forty years ago, Ludwig Feuer and Gerald Holton traced this pedigree using similarities between texts of the philosophers and Bohr's quantum philosophy. Feuer's close study identified the Ekliptika, Høffding, James, and Kierkegaard as sources of Bohr's characteristic concepts of discontinuity, renunciation, and subjectivity, and linked them to the quantum leaps in the hydrogen atom.[65] Holton emphasized James' pragmatism as mediated through Høffding's pedagogy, Christian Bohr's concern with vitalism, and Kierkegaard's leaps and stages, and connected them with the principle of complementarity.[66] Other analysts, notably the late David Favrholdt, defending what they took to be a higher intellectual descent, downplayed these similarities as vague and imprecise, and insisted that, since Bohr never studied philosophy systematically, his originality came unpolluted by philosophical schools. On this interpretation, the quantum atom and complementarity arose strictly from physics.[67] The most that Favrholdt and, equally authoritatively, Bohr's long-time collaborator Léon Rosenfeld, allow is a link to Møller.[68]

The newly available correspondence between Bohr, Margrethe, and Margrethe's mother confirms the conjecture of Feuer and Holton, and, more radically, allows a more stringent application of it to an earlier phase of Bohr's creativity. Høffding and Kierkegaard and also Høffding's admirer William James can be glimpsed behind the quantum atom. In the creation of the trilogy, physics did not precede physics or physics philosophy: they were inextricable. "Philosophy" here must be understood as a humanistic amalgam of philosophical questions and literary expressions of their solutions. Favrholdt rightly rejected fathering Bohr's creative thought, idea for idea, on a formal philosophy.[69] Bohr himself was unable to say how he came to take up philosophical problems. "I do not know. It was in some way my life. . . .It was a natural thing for me to get into a problem where one really could not say anything from the classical point of view."[70] We can do a little better in identifying sources of his philosophical interests. Besides possible impulses from his Jewish heritage, gleanings from the discussions of the four professors, Høffding's lectures and Kierkegaard's *Stages*, we can also notice Bohr's familiarity with classical literature dealing with deep questions, especially Goethe, a favorite of his father, and Ibsen, a favorite of Høffding, among the unusual furnishings of his mind.[71]

Soon after his arrival in Cambridge and still full of confidence, Bohr attended a luncheon given by the mathematician G.H. Hardy, to whom Harald had provided an introduction. Niels took the occasion to divert the company with his notions of the

[63]Fenger, in Hertel and Kristensen, *Activist* (1980), 50-2; Høffding, in Murchison, *History* **2**, 197 (1932).
[64]Andersen, in Møller, *Skrifter* (1930), 1, viii.
[65]Feuer, *Einstein* 111, 114–15 (1974), 122, 134–6, 139–44.
[66]Holton, Daedalus, 1970, 1040–44.
[67]Favrholdt, *Filosoffen* (2009), chapt. 6.
[68]Favrholdt, *Bohr's philosophical background*, 35–6 (1992), and (for Rosenfeld), Holton, Daedalus, 1970, 1052n24.
[69]Favrholdt, in *CW* **10**, 301–3, and, to overkill, in Favrholdt, *Bohr's philosophical background* (1992), 22-31, 74-118.
[70]Bohr, *Interview*, 76, 77 (1962).
[71]Høffding, a great reader himself, complained that most students of Bohr's generation did not know the great writers; Rindom, *Samtaler*, 52, 57 (1918), and *Høffding*, 84 (1913); AH, 106–9.

nature of truth. Unfortunately he did not report any more about the discussion than that those present declared they had not heard the like before.[72] Since he did not change his mind easily, we may assume that he elaborated for the mathematicians the doctrine that he expressed in a letter to Margrethe: truth is not singular, but comes in hierarchical multiples. There are the truths of great literature, which are greater, because "more universally human," than the truths of a sermon, and closer to the "so-called scientific truths, which again are of a somewhat different kind." This generous view, in which many special truths of different sorts make up a total vision (though never the total Truth), was not a gambit got up for Cambridge conversation. "[I]t is something I feel very strongly about; I can almost call it my religion, that I think that everything that is of any value is true."[73] Or, as James put it, multiple truths are the truth: "The whole notion of the truth is an abstraction from the fact of truths in the plural."[74]

Høffding's epistemological teachings ended in much the same place: no single truth can capture a domain, for as analysis is pushed ever further, an inevitable, irremediable, inaccessible residuum will appear that does not, will not, yield to rational analysis. This was the proposition that Bohr told Margrethe's mother he could prove logically. He did not say that he had taken it, and his pleasure over the existence of an irrational residuum, from that "good pluralist and irrationalist," Harald Høffding.[75] Enthusiasm over the necessity of renouncing the search for a theory of everything marked Høffding's modest epistemology; as Rubin recalled, "this state of affairs caused him great and profound satisfaction," for, like Bohr, he regarded its contrary, in which everything would stand revealed, as the destruction of "an essential condition for the value of human life."[76] One of his students suggested that the motto for his teaching should be a line from Goethe he often quoted, "nie geschlossen, oft geründet," which in context signifies "tireless searching, firmly founded/never ended, often rounded." Bohr knew the entire verse by heart, perhaps from his father, and quoted it to Margrethe, who suggested that it be "our poem."[77]

Høffding had begun his university studies in neither philosophy nor physics, but in theology, and it took him longer than it would take Bohr to break with organized religion. But after a long internal fight guided by Kierkegaard's similar struggle he decided, as Bohr would, that he could not live his life "by the ideals and commandments of religious ethics" and looked to philosophy to develop a more embracing humanism.[78] This did not solve but merely defined his ongoing problem, which he classed as the "greatest challenge of science: to understand the human condition in ever greater depth and over a continually broadening horizon."[79] "Even one who is of the opinion that the times of religion have gone by – an opinion which must be epistemologically, psychologically, and ethically grounded if it is to be more than an assertion or a wish – will still feel the necessity of finding equivalents for

[72]Niels to Margrethe, 12 Dec 11, AH, 39–40, 173.
[73]Niels to Margrethe, 15 Jan 12, AH, 174.
[74]James, *Pragmatism*, 92 (1907).
[75]The evaluation of James, after hearing Høffding lecture in 1904; James to F.C.S. Schiller, [1904], in James, *Letters* **2**, 216 (1920).
[76]Rubin, *Experimenta*, 20 (1949).
[77]Jørgen Jørgensen, quoted by Witt-Hansen, Dan. yearb. phil. **17**, 46 (1980); Rindom, *Høffding*, 86–7 (1913); AH, 61; Goethe, Gott und Welt: "Weite Welt und breites Leben / Langer Jahre redlich Streben / Stets geforscht und stets gegründet / Nie geschlossen, oft geründet."
[78]Høffding, *Kierkegaard*, 3–4 (1896); Rindom, *Høffding*, 21–31, 68–9 (1913).
[79]Hansen, *Høffding*, 31 (1913).

the loss of belief in those goods which the vanishing of religion entails."[80] And so Høffding defined the great religious problem as (in the words of James) "the ultimate 'conservation of values,' or of what has value."[81]

Bohr gave expression to the same program when he assured his future mother-in-law that he believed in many things: "in the goodness and love of human beings, for that I have experienced;" "in the duties of a human being, although I cannot say exactly what they are;" and "in so many many other things that I do not understand." How can these things be justified, grounded, in absence of religion? Bohr could only hope, "with all my soul" and without supernatural help or threats, that he could stay true to his ideals of "the good and great and true."[82] This was a moral, if not a philosophical solution, to the great problem, which, as he knew from Høffding, could not be solved.[83] That did not condemn either of them to relativism. Bohr judged that he believed much more than his in-laws did, "for I believe in the happiness and meaning of life." They in contrast believed that "the salvation of a human being is contingent upon whether he can or will believe in three or four propositions without content or meaning."[84]

For Høffding, free inquiry in the religious sphere was the pre-eminent means for awakening and encouraging thought. "He to whom the problem [of religion] does not present itself has of course no ground for thought, but neither has he any ground for preventing other people from thinking."[85] Høffding's evenhanded consideration of religion persuaded his students, "for whom his lectures were the experience of their university years," and worried their parents, who feared, rightly, that he might dissolve their traditional beliefs.[86]

Høffding put an extravagant value on intellectual life. So did young Bohr ("it is the most valuable and only thing I possess"), who hoped to qualify in the only class of scientist that, according to his professor, required true scientific culture. These were the creators of new theories. Most scientists, according to Høffding, either applied others' ideas or, lower yet in the intellectual order, just followed, in "pure and simple acceptance, and trust in legitimacy and tradition." "One of the wisest, as well as the most learned of modern philosophers," Høffding exemplified the highest and most general scientific culture. That was Rubin's opinion: "His work presents a singular mixture of a strict scientific spirit and a personal, almost an artistic tendency. A characteristic trait is his appreciation of the feelings attendant on the deepest scientific research." He was a serious person. "I've never really been young," Høffding acknowledged in reply to Georg Brandes' accusation that he had always been forty; but then, intellectually speaking, he never grew old.[87]

It is not surprising that the clever members of the Ekliptika club, energized by such a teacher as Høffding, aspired to and reached the highest circles of Danish academic life. Students spoke of him as "the philosopher" and continued to read him after graduation, inspired by his liberal, high-minded formulation of significant

[80]Høffding, *Problems*, 180 (1905); cf. Rubin, *Experimenta*, 25 (1949), and Høffding, Int. jl ethics **22**:2, 150 (1902).

[81]James, in Høffding, *Problems* (1905), xiii; cf. ibid., 176–7, and Rindom, *Høffding*, 85–6 (1913), and Hansen, *Høffding*, 31 (1923).

[82]Bohr to Sophie Nørlund, 1 May 12, AH, 77.

[83]Høffding, *Problems*, 186 (1905).

[84]Niels to Margrethe, 1 May 12, AH, 78.

[85]Høffding, *Philosophy*, 3 (1906).

[86]Rindom, *Høffding*, 70, 79 (1913).

[87]Niels to Margrethe, 19 Jul 12, AH, 92 (first quote); Rubin, *Experimenta* (1949), 27, 28, 22 (second and fourth quotes); James, in Høffding, *Problems* (1905), v (third quote); Rindom, *Samtaler* (1918), 64 (fifth quote).

insoluble problems if not by his larger constructions.[88] A description by Rubin of a paper he wrote for Høffding's seminar may indicate the level of problems and solutions presented to the Ekliptika. It concerned the character of patriotism. Rubin's approach adumbrates the "aspective view of wholes" that informed his later psychology. Aspects are not elements, and patriotism is not reducible to them.[89] I will resist the temptation of paralleling Rubin's wholes and Bohr's quantum. We may glimpse another residue of Ekliptika discussions in Niels' remark to Harald, "sensations, like cognition, must be arranged in planes that cannot be compared." The remark, evidently made in the context of an ongoing exchange, was offered jocularly to justify Niels' unwillingness to declare which of three presents made him happiest.[90]

Høffding stayed in contact with the Bohr family after Christian Bohr's death in 1911. In his old age his "good friend Niels Bohr" would visit him to talk about physics and philosophy, and read from their favorite poets, for "Niels Bohr is not only a great physicist, but also is interested in philosophy and literature."[91] Høffding was able to make use of Bohr's theory of the periodic table in a widely published essay on the concept of analogy and Bohr, returning the compliment, credited Høffding with "ideas that helped physicists to understand their work."[92] On Høffding's death, Bohr succeeded him in the "Aeresbolig," the villa left by the founder of the Carlsberg Brewery as the home of the greatest intellectual among the Danes as determined by the Danish Academy of Sciences. The succession might serve as a symbol of Bohr's place in Danish philosophy and culture. "Symbolisation is necessary if you want to express the latest results of [scientific biography]."[93]

It is time to prove "logically" that human beings can never know everything. Let us begin with physics. The great ones, Maxwell and Hertz, held that our theories are mental representations or sets of symbols, neither unique nor comprehensive. To be comprehensible, however, all posit continuity of action, as in dynamics, which we follow by continuity of thought; that is our mode of understanding. "The great question is, whether the idea of the continuity of motion or activity can be carried out in all spheres." If not, room opens for "an irrational relationship between Being and our knowledge." Consideration of the concept of causality brings us to the same place. Though it is riddled with logical difficulties, we cannot do without it; "for us, existence can never be absorbed into thought without remainder."[94] But this is only foreplay. Our incompetence can be brought home fully and forcefully by considering that our knowledge supposes a clean division between the subject (the observer) and the object (the observed). This is an indulgent delusion. Object and subject mutually determine one another: a pure subject is as illusory as a thing-in-itself. Not only is there no pure case, but no place to stop: a fresh subject S_O contemplating an object O_S creates the subject/object S_1/O_1, which, by interaction of its parts, produces

[88] Jacobsen and Brønsted, "Inledning," in *Relig. brevv.* (1964), vi, xii, xvi, xviii; Jacobsen to Søren Alkaersig, 27 Feb 17, ibid., 214, and Niels Møller to Jacobsen, 3 Jan 18, ibid., 261.

[89] Rubin, *Experimenta* (1949), "Preface."

[90] Niels to Harald, 26 June 10, in *CW* **1**, 513.

[91] Høffding to Meyerson, 12 Feb 24 (first quote), 23 Apr 26, 13 Apr 28 (second quote), 7 Oct 29, in Brandt et al., *Correspondance* (1939), 70, 123, 156, 169.

[92] Høffding to Meyerson, 20 May 23, ibid., 51, re Høffding, *Der Begriff der Analogie* (1924); and 30 Mar 28, ibid.,149, re Bohr's éloge on Høffding's 85th birthday (*CW* **10**, 308–9).

[93] Høffding to Meyerson, 30 Dec 26, in Brandt et al., *Correspondance*, 131 (1939); the original has "physics" for the words in brackets. Høffding was second choice for the honor of first inhabitant of the Aeresbolig after Thomsen, the philologist in the Høffding-Bohr quartet, declined because of failing health. Rindom, *Samtaler*, 72–4 (1918).

[94] Høffding, *Problems*, 90–2, 93–4 (1905) (first quotes), 94–106, 107 (third quote).

S_2/O_2, and so on. "Here again we run up against the irrational and here perhaps we see most clearly how inexhaustible being is in comparison to our knowledge."[95] There is no reason for despair in the realization that human beings cannot create "an exhaustive concept of reality;" for it is just in "the irrationality in the relation between thought and reality [that]...the possibility of progress lies."[96]

Kierkegaard says the same things even better, as Bohr could have read in Høffding's succinct summary of the philosophy of "the greatest of our thinkers." According to Kierkegaard's account of the subject-object dilemma, it is logically impossible for us to create a complete account of Being because our knowledge and experience grow and change; and as we are part of the Being we are trying to capture in thought, we are attempting to grasp something unformed or continually forming. (Kierkegaard snickered that academic philosophers had missed this point because they are such non-entities that they excluded themselves from existence in general.[97]) This is the problem of the Subject altered by the Object; a problem that, to continue the regress, can be followed into Møller's story of the student addicted to thought who drives himself into intellectual impotence by thinking about himself thinking about a second self thinking..., and into physical impotence by finding no sufficient reason to perform an action immediately or a second later, or a second after that, and, hence, at any time at all. And worse, since every thought must have a direction, which obviously must be known before the thought it directs, a decision that seems a minute's work presupposes an eternity.[98] Bohr thought this story so expressive of the problems of quantum physics and the Danish way of handling them that he later urged it on all his foreign students as soon as they knew enough of the language to read it.[99] For it presented not only the problem of the division between subject and object, but also the need sometimes to break off a logical line of thought arbitrarily in order to progress. As the stymied Danish student discovered in his lengthy ruminations, "It is the reality of time that makes the world irrational for us."[100]

Continuing his rendering of Kierkegaard, Høffding declared that whatever understanding we achieve can only be retrospective. As he put the point in a lecture to James' students at Harvard in 1904, "we live forward but understand backward." Not everything lends itself to backwards comprehension, however; we will never be able to explain how we can understand retrospectively the necessity of what was open-ended prospectively.[101] This was to phrase the problem of free will in precisely the terms in which Bohr later approached it, by the doctrine of multiple partial truths: we are free in prospect, bound in retrospect. "[A] situation that calls for a description of our feeling of volition and a situation demanding that we ponder on the motives for our actions have quite different conscious contents."[102] Or, as Høffding put it more clearly and distinctly in his textbook on ethics in 1897, investigating experience as it occurs would be like standing on your head and legs at the same time.[103]

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[95]Ibid., 107–11, 112–13 (quote).

[96]Ibid., 114–15; Høffding, Int. jl ethics **22**:2, 149 (1902), resp. Cf. Høffding, Jl phil. psych. sci. methods **2**, 88–9 (1905).

[97]Høffding, *Problems*, 112–13 (1905).

[98]Møller, *Skrifter* **1**, 292–3, 326 (1930).

[99]AH, 107; Feuer, *Einstein*, 126–31 (1974).

[100]Høffding, Jl phil. psych. sci. methods **2** , 88 (1905).

[101]Høffding, *Kierkegaard* (1896), 2 (quote), 63, 66.

[102]E.g., *CW* **10**, 143, 159–60, 200 (quote), 279.

[103]Høffding, in Murchison, *History* **2**, 203 (1932), with reference to complementarity.

Kierkegaard regarded his main task as criticism, as raising difficulties about accepted beliefs. Among his preferred targets was the assumption that we can make "a smooth and continuous connection [among the parts of] our knowledge." That was wrong both intellectually and morally. "It is only reprehensible laziness or impatience that makes us believe that there must be something complete and closed."[104] Bohr's letter to Margrethe's mother is in close harmony with this unfriendly view of dogmatic systems. Bohr also would have resonated with Kierkegaard's claim to the role of universal critic. For if there was anything at which Bohr excelled as a young man it was criticism.[105]

The centerpiece of Høffding's précis of Kierkegaard is the notion of distinctive and even discontinuous stages or types of civilized life. Bohr's blood boiled (as it often did when reading good literature or writing to Margrethe) over Kierkegaard's presentations of this theme in *Stages on life's way*. Here we are on unusually solid ground because in 1909 Niels sent his copy of the book to Harald as a birthday present with a commendation that reads as follows: "It is the only thing I have to send; nevertheless, I don't think I could easily find anything better. . . . I think absolutely that it is about the most beautiful thing that I have ever read."[106] He sent the book to Harald from a parsonage to which he had withdrawn from the bustle of quiet Copenhagen to prepare for his master's thesis and examination. It was just the place for a romantic intellectual. "I walk here in solitude [he wrote Harald] and think about so many things." He thought about physics, of course, and mathematics and logic, but also about the problem of cognition, the stages of life, the nature of the good.[107] The episode meant something to Bohr as he could still relate it in accurate detail many years later. "[Kierkegaard] made a powerful impression on me when I wrote my dissertation at a parsonage on Funen, and I read his works day and night. . . His honesty and willingness to think the problems through to their very limit is what is great. And his language is wonderful, often sublime."[108]

Kierkegaard's insight into the human condition was so deep that he had to divide himself into a dozen different personae to do justice to it. These personae appear in his books as characters and on his title pages as pseudonyms. He needed six of them to convey the truths in *Stages*. The earliest stage, the aesthetic, which for some people lasts a lifetime, is a period of carefree experimentation, of flitting from one experience or idea to another. Kierkegaard depicts it through speeches given by four of his avatars at a symposium on love, life, and the universe. Each says something true, though his statement conflicts with what the others say. Another avatar, a self-satisfied judge, sets forth the merits of a good marriage, the essence of the second or ethical stage. The judge's wife was patient, understanding, supportive, protective, enabling him to reach the highest level his talents and training permitted; neither he nor she could achieve as much apart as they did by pooling their complementary qualities; each contributed an equal share to the truths of married life. Bohr needed such a partner more than most men. As for the third and final stage, the religious, it can be reached only by a leap of faith, which, as we know, was a quantum jump that Bohr made in the opposite direction.

[104] Høffding, *Kierkegaard*, 57, 63 (1896).
[105] AH, 128–9, 135, 154.
[106] Niels to Harald, 20 Apr 09, *CW* **1**, 501.
[107] Niels to Harald, 20 Apr 09, *CW* **1**, 501 (solitude); 17 and 27 Mar 09, ibid., 499 (logic); 26 Apr 09, 503 (notes); 26 June 10, 513 (cognition); 9 June 09, 505 (mother as amanuensis).
[108] Remarks by Bohr in 1933 recorded by J. Rud Nielsen as quoted by Holton, Daedalus, 1970, 1053 n47. Cf. Feuer, *Einstein* (1974), 122, re quantum jumps and "Either/Or."

Another of Kierkegaard's personae made a perfect model for a romantic young critic walking in solitude around a country personage. This was Johannes Climacus, who had a passion for thinking so intense that he could not think about girls. "In love he was, madly in love, but with thought, or rather with thinking." He worried constantly about the meaning of the key of philosophy, the slogan *de omnibus dubitandum est.* Having a "romantic soul which always looked for difficulties," that is, being a consummate critic, Climacus managed to prove that the foundational principle, "modern philosophy begins with doubt," to which every philosopher from Descartes on had ascribed some meaning, did not mean anything at all.[109] And if it did mean anything, it would exterminate the race of philosophers, since every student would be obliged to doubt the words of his teacher, and each generation would slay its predecessor. And so poor Climacus never advanced even to the threshold of received philosophy. "He became more and more retiring, fearing that thinkers of distinction might smile at him when they heard that he too wanted to think."[110]

2 Transitions: August 1912 – February 1913

With Bohr's return to Copenhagen the main source of information about his activities, his correspondence with his family, dries up apart from a few letters to Harald, who was studying in Göttingen. We know that his time was occupied in setting up house and teaching at the university, where he obtained the junior post released by Knudsen's succession to Bohr's Doktorvater Christian Christiansen. Bohr gave an ambitious course on thermodynamics making use of his detailed reading of Poincaré's text and did experiments for Knudsen on friction in gases at low temperatures.[111] Teaching and laboratory work took up so much time that he could not finish the development of the ideas he had sketched the previous July. Eager to fulfill his promise to send Rutherford a paper and to show that he belonged in the rarified group of creative scientists, Bohr asked to be relieved of his duties and retired with Margrethe to the countryside to write. The lengthy result of this rustication eventually appeared as the second and third parts of the trilogy of 1913. They remain within the range of topics touched on in the Rutherford Memorandum of July 1912.

New stimuli

Two stimuli from outside Rutherford's circle prompted Bohr's transition from the qualitative model of the later parts of the trilogy to the famous first part on the spectrum of hydrogen. The order of publication of the parts thus hides the order of their conception: the applications to the "constitution of atoms and molecules," to use the title carried by the entire sequence, did not extend the principles apparently established by agreement between theory and measurement of hydrogen's Balmer lines, but antedated the systematic consideration of spectra.[112] The hastily composed Part 1 contains contradictions and redundancies that Bohr might have removed had he taken more time. The rush was fortunate as the resultant blemishes transmit precious material to the historian, and perhaps, the psychologist.

[109] Kierkegaard, *Climacus*, 103 (first quote) (1958), 116 (second quote), 126, 140 (third quote).
[110] Ibid., 138, 115 (quote).
[111] Bohr, *Interview* 53 (1962); HK, 255.
[112] HK, 255–6.

The first of the two external stimuli that prompted Bohr to consider spectra came as a shock. The instigator was a Cambridge mathematician, John William Nicholson, four year's Bohr's senior, who taught at the Cavendish during Bohr's sojourn there before moving to the chair of mathematics at King's College, London. Just before Bohr's arrival, Nicholson published a lengthy deduction of the number of free conduction electrons in various metals. Most of Nicholson's assumptions agreed with Bohr's except that Nicholson added the contributions of "vibrating" positive metal ions to the total current, which made possible connections with the metal's index of refraction and dielectric constant. Finding that the number of free electrons in magnesium was about 3.14 times the number of magnesium atoms, he argued, characteristically, that magnesium might have three electrons or, better, seven such atoms, forming a "magnesium complex," might have 22 electrons available for conduction. Bohr's verdict on this analysis: "perfectly crazy." Nicholson not only relied on incorrect calculations made by others, but also, in applying them, missed the same point that Thomson and Darwin did: he did not take into account that the period of the excitation (in this case of visible sodium light) had the same order of magnitude as the intervals between collisions of the electrons with the metal molecules. Bohr hunted up Nicholson to tell him about his errors. "[H]e was extremely kind, but with him I'll hardly be able to agree about much."[113]

This remark may also have referred to an even crazier piece that Nicholson had in press. It dealt with the structure of atoms. Following a nineteenth-century precedent, particularly the speculations of Norman Lockyer, Nicholson imagined that the chemical elements had evolved in stars as compounds of still more elementary substances. Nicholson modeled these supposititious building blocks as one-ring nuclear atoms. Although he knew about Rutherford's model, he credited Thomson with the basic idea of a positive charge proportional to its volume. Thus the radius a of a charge ne would be proportional to $n^{1/3}$ and Nicholson's proto-atoms would be one-ring versions of Thomson's with the electrons circulating outside the positive sphere rather than within. Nicholson knew perfectly well that this little difference brought big problems. He sidestepped the radiation problem by disallowing the one-electron case; as Larmor and Thomson had pointed out, by placing two or more electrons symmetrically so that their accelerations summed to zero, radiation loss from the ring can be made very small.[114]

As for mechanical stability, Nicholson ignored the troublesome planar vibrations and attended only to stable oscillations perpendicular to the ring plane. The outcome of his calculations was astonishing. The perpendicular oscillations of the 4-ring uratom "nebulium [*Nu*]" accounted for ten unattributed lines in the spectra of nebulae with an accuracy of 1 part in 10,000, and the 2-ring and 5-ring versions ("coronium" and "protofluorine [*Pf*]") accounted for fourteen lines in the solar corona to 1 part in 1000. As for the 3-ring, Nicholson identified it, or a polymer of it, with ordinary hydrogen.[115]

Esse est percipi: Since *Nu* and *Pf* glow they must exist. Consequently they have weight. Nicholson assumed that their mass was entirely electromagnetic and that, therefore, ignoring the very slight contributions of the electrons, he could assign his models masses proportional to n^2e^2/a (the electromagnetic mass of a sphere of

[113]Nicholson, PM **22**, 245, 263, 266 (Aug 1911); Bohr to Oseen, 1 Dec 11, in *CW* **1**, 423, 427.
[114]Heilbron, in Weiner, *History*, 46–7, 54–5 (1977).
[115]Nicholson, PM **22**, 865, 868 (Dec 1911). Nicholson published the details of the spectral matches in the *Monthly notices* of the Royal Astronomical Society; McCormmach, Arch. hist. exact sci. **3**, 176–9 (1966), and HK, 258–62.

radius a as calculated by Thomson), which, with $a \approx n^{1/3}$, made mass proportional to $n^{5/3}$. He then had the "atomic weights" of his uratoms in terms of hydrogen's, for example, $A_{Pf} : A_H = (5/3)^{5/3}, A_{Nu} : A_H = (4/3)^{5/3}$. Taking $A_H = 1.008$ (oxygen = 16), Nicholson made out that, since the weights of nebulium and proto-fluorine summed to 3.99, $He = Nu + Pf$. The rest is numerology. It will be enough to state that radium is $H_{30}Pf_{30}He_{25}Nu_{16}$ or, as Nicholson wrote it to bring out its relationship to other alkali earths, $8[He_2Nu_2(PfH)_3]2[He_2(PfH)_3]He_5$. Nicholson obtained this gigantic formula, which gives $A_{Ra} = 226.8$ in happy agreement with the experimental determination of 226.4, by adding an alpha particle to his construction for radium emanation "on the assumption that its α particle is helium."[116] But are not all alpha particles helium nuclei? "There is, in fact, strong reason to doubt this view." To be sure Rutherford and Royds had found helium lines emanating from a vessel free from helium in which they collected alpha particles. But the spectrum they recorded included a few lines not attributable to helium, for which Nicholson proposed HNu or Nu_2.[117]

Radioactivity abounds in complications that Nicholson's complicated formulas could help resolve. For example, a daughter radioelement descended via beta or gamma emission would have the same atomic weight as its parent and the same constituent uratoms differently arranged. That is probably the case (Nicholson suggested) with the emanations of radium and thorium. But not with actinium emanation, to which Nicholson ascribed a much lower atomic weight and a descent from actinium via "α particles" that are Nu_2 rather than helium. (In contrast to both Thomson and Rutherford, Nicholson could explain alpha emission easily since the expelled particle pre-existed as a distinct entity in the disintegrating atom.) Actinium itself probably comes from the breakup of uranium into large pieces. "A gas like neon may be an α particle from certain kinds of matter, and it is probable that all the inert gases are waste products of this nature, which have accumulated in the atmosphere."[118] From which it appears that Nicholson's imagination anticipated the concepts of isotope and fission, and the Gaia hypothesis.

At the time of his unsatisfactory discussion with Nicholson, Bohr wrote his colleague Carl Wilhelm Oseen (virtually the only established theoretical physicist in Scandinavia) that he was "very enthusiastic about the quantum theory (I mean its experimental side)."[119] Bohr did not pursue the subject until he engaged with Rutherford's atom and then he found that Nicholson had preceded him. It had occurred to that man of imagination that Planck's quantum might be found in his uratoms. Possibly the spur to look came from some knowledge of the Solvay proceedings or from Poincaré's several statements of his proof, prompted by the Solvay discussions, that the derivation of Planck's radiation formula required some discontinuity. An obvious place for Nicholson to look for h was the quantity T/ω, where T is the total kinetic energy of the electron ring and ω the orbital frequency of its electrons. By matching the frequencies of the transverse vibration of the ring (which are functions of the frequency of orbital motion) with the nebular and coronal lines, Nicholson knew ω for Nu and Pf; and from ω he knew a, the ring radius, by ordinary mechanics. He could therefore calculate $T/\omega = 2\pi^2 ma^2\omega$. It fell out close to $5h$ per electron for Pf, or $25h$ for the entire ring. He must have been ecstatic to find

[116] Nicholson, PM **22**, 870, 873–4, 885 (Ra) (Dec 1911).
[117] Ibid., 875. Rutherford and Royds, in Rutherford, *Papers* **2**, 134–5 (1962).
[118] Nicholson, PM **22**, 867, 880–1, 883 (quote), 884, 888 (Dec 1911).
[119] Bohr to Oseen, 1 Dec 1911, in *CW*, **1**, 431.

that singly and doubly ionized forms of *Pf* gave, for the ring as a whole, $T/\omega = 22h$ and $15h$, for, continuing the harmonic series, a *Pf* ring with two electrons would have $T/\omega = 13h$, with one electron $7h$, and with none, 0, "as would be expected." Now $2\pi^2 ma^2\omega = \pi G$, where G is the angular momentum per electron. Nicholson therefore proposed that any one of his uratoms could radiate a set of lines that originated in electrons whose angular momenta differed from one another discontinuously.[120] This is not quite the quantum rule that later prevailed, since Nicholson had $G = ph/\pi$, where in general p is a rational fraction, whereas the Bohr condition has $G = nh/2\pi$, where n is an integer. As will appear, Bohr's efforts to secure the factor 2 in this equation provoked the most revealing indications of his thought.

Nicholson gave a synopsis of his atomic theories in September 1912 at the annual meeting of the British Association for the Advancement of Science, thereby opening a discussion of the problems of atomic structure that continued at the Association's meeting in 1913. In the exchange after Nicholson's synopsis, which Bohr could have read in *Nature* in December 1912, he was asked how his uratoms could give rise to series spectra. He replied that they did not and could not; series arise from more complicated structures, chemical atoms, which, with the possible exception of hydrogen, have more than one nucleus. He held that series spectra cannot be modeled dynamically, only kinematically, and gave as an example a generalized Balmer formula written as $\lambda = \lambda_0 n^2/(n^2 - a^2)$.[121] If Bohr saw it written thus, in terms of wavelength λ and a disposable constant a, he saw nothing in it of interest to him. Nicholson's reply to the question on spectra prompted Lord Rayleigh to call attention to a distinction almost invariably overlooked, "the difference between the vibration in the atom and that received by the observer," which he illustrated by an acoustical example of uncertain relevance. He went on to remark, in the context of a report by F.A. Lindemann on the application of quantum theory to specific heats at low temperatures, that he thought there was something in the theory, although "it implies the extraordinary result that when two molecules meet they may not take up motion because it is too small to be taken up at all."[122] Two prescient remarks.

Rutherford responded to Lindemann, and to "foreigners" (although Lindemann was an Englishman) in general, that they "seemed to be content without realizing a practical model or mechanism of the process they assumed to take place." He had in mind the reproduction of the experimental curves of specific heat against temperature by adding together two exponentials of the type that Einstein had derived from Planck's theory of radiation for application to vibrations of solids. Rutherford: "A double exponential equation could be fitted to anything." A similar objection might have been made of Nicholson's formulas for the chemical elements. After this sure-footed start, Rutherford described his unfortunate theory of the origin of beta and gamma rays, which assigned an extra-nuclear origin to both.[123]

The threat of Nicholson's work to Bohr's nascent quantum atom was plain enough. Nicholson's atom, like Bohr's, was nuclear and quantized; Nicholson gave abundant quantitative results, Bohr virtually none; Nicholson's could radiate, Bohr's could not; and Nicholson had priority. "I thought at first," Bohr wrote Rutherford, "that the one or the other necessarily was altogether wrong."[124] If he had known

[120] HK, 255–6; McCormmach, Arch. hist. exact sci., **3**, 169–70 (1966).
[121] Nature **92**, 424 (12 Dec 12).
[122] Rayligh, ibid., 424, 423.
[123] Rutherford, ibid., 423, 425, and (on beta and gamma rays), *Papers* **2**, 286–7 (1962) (text of Aug 1912).
[124] Bohr to Rutherford, 31 Jan 13, in *CW* **2**, 579.

about Nicholson's publications as they came out, Bohr might have been distressed, even depressed; but he could console himself with the thought that Nicholson could write nonsense, and, in the familiar environment of Copenhagen at Christmas time, he took an optimistic view. "Nicholson seems to be concerned with the atoms while they radiate," whereas Bohr dealt with "the final, chemical state of the atoms."[125] Perhaps he was assisted in this assimilation by Thomson's suggestion that an electron could emit an entire spectrum as it settled down in the atom if it passed through a series of spherical shells in each of which it could revolve long enough to emit a spectral line.[126] Apparently Bohr was too full of good cheer and domestic bliss to worry that there was no place for nebulium and protofluorine in his universe, or for atoms with multiple nuclei.

As late as 31 January 1913, when he wrote Rutherford about his progress, Bohr entertained the same view of the relationship between his model and Nicholson's that he had worked out in Christmas charity. One related to atoms during their formation, to the states "in which the energy corresponding to the lines in the spectrum characteristic for the element in question is radiated away;" the other, Bohr's, dealt with atoms in their "permanent (natural)" state. Nicholson's states could occur only where atoms continually break up and reform, as in discharge tubes and nebulae. Bohr's state in contrast was in agreement with experimental facts pertinent to atoms when "permanently" arranged. With these observations, Bohr shelved Nicholson and returned to his earlier concerns. "[T]he considerations sketched here [his letter to Rutherford continued] play no part of the investigation in my paper. I do not at all deal with the question of calculation of the frequencies corresponding to lines in the visible spectrum."[127]

Soon after sending this letter with its awkward compromise to Rutherford, Bohr received the second stimulus to adapt his developing model to emit and absorb radiation. This time chance favored the prepared mind. The chance was the question, put to him by the spectroscopist Hans Marius Hansen, how the quantum atom handled series spectra. Bohr replied to the question much as Nicholson had done at the British Association meeting a few months earlier: the complicated spectral formulas seemed beyond the reach of his theory. Hansen pointed out that nothing could be simpler than the Balmer formula, which he presented (or Bohr otherwise saw) as a relation of frequencies, not wavelengths:

$$\nu_n = R(1/2^2 - 1/n^2). \tag{1}$$

As soon as he saw this expression, Bohr recalled, he understood its significance.[128] For multiplying both sides of equation (1) by h in the spirit of Planck, he would have made the Balmer formula a statement of the conservation of energy. The emitted radiant energy $h\nu_n$ derived from a loss of internal energy by the atom; this energy did not come out through continuous vibrations perpendicular to one of Nicholson's radiating rings, but discontinuously, in a transition from an orbit more distant to one closer to the nucleus. Still, Nicholson might have seized on a useful partial truth, if not about spectral emission, then perhaps about the mechanism of dispersion. It was hard to tell. "I felt that maybe one could not say that it was untrue."[129]

[125] Niels to Harald, 23 Dec 12, in *CW* **1**, 563, with the reading "classical" corrected to "chemical."

[126] Thomson, *Corpuscular theory*, 156–61 (1907), which suggests two clever implausible ways of establishing the shells.

[127] Bohr to Rutherford, 31 Jan 13, in *CW* **2**, 579–80.

[128] HK, 264–6.

[129] Bohr, *Interview*, 15–16 (quote), 44 (1962); PM **26**, 6–7, 23–4 (1913), in *CW* **2**, 166–7,183–4. Bohr took some

Outcomes

Consideration of Nicholson's radiation theory had prepared Bohr for the acceptance of states above the ground state characterized by an integer n that defined the number of quanta each electron in the higher state possessed. Quanta of what? Bohr did not take over Nicholson's answer, "angular momentum," or perhaps he did, only to find out that it gave a value of R (in equation (1)) in disagreement with experiment. As we know, Nicholson found that angular momentum $G = ph/\pi$; but, as Bohr soon discovered, he needed to make $G = nh/2\pi$. He may have come to this realization by generalizing his condition of the ground state to $(T/\omega)_n = K_n$, and deduced that $K_n = nh/2$ by equating Rh/n^2, which, from his insight into the Balmer formula, he recognized as the negative of the energy of the nth state, with T_n.

Here is how it could have been done. From $(T/\omega)_n = K_n$ and the classical force balance for an electron describing a circular path of radius a around a nucleus of charge Ze, that is,

$$Z^2e^2/a^2 = 4\pi^2 m\omega^2 a, \tag{2}$$

it follows, with a little algebra, that

$$a_n = K_n^2/\pi^2 mZe^2, \quad \omega_n = \pi^2 mZ^2e^4/2K_n^3, \quad T_n = \pi^2 mZ^2e^4/2K_n^2. \tag{3}$$

If $T_n = Rh/n^2, K_n = nK$; and if, as Bohr assumed all along, K is a submultiple of h, say αh,

$$R = \pi^2 mZ^2e^4/2h^3\alpha^2. \tag{4}$$

Using the numbers Bohr employed in Part 1 of the trilogy, $e = 4.7 \times 10^{-10}$, $e/m = 5.31 \times 10^{17}$, $h = 6.5?10^{-27}$, and the measured value for R, he would have found that $\alpha = 1/2$. The defining equation of the nth state therefore became

$$T_n = nh\omega_n/2. \tag{5}$$

Reversing the procedure, beginning with (5) and (2), Bohr had the now famous formula,

$$R = 2\pi^2 mZ^2e^4/h^3 = 3.1 \times 10^{15}, \tag{6}$$

in good agreement with the measured value for R for hydrogen, $R_H = 3.290 \times 10^{15}$. It remained only to justify equation (5).

Bohr gave four distinct and mutually contradictory justifications or groundings in the hastily written Part 1 of his trilogy. The first two invoke the analogy to Planck's theory and the formation of helium from an alpha particle. But now the conditions based on the analogy contained a running integer n and a half-integral multiplier. Bohr proposed to equate the spectral frequency ν_n with the average orbital frequencies involved when the bare hydrogen nucleus captures an electron into the nth "stationary state." Taking the orbital frequency of the unbound electron ω_∞ as 0, Bohr had $\nu_n = 1/2(0+\omega_n) = \omega_n/2$. That would make the fundamental postulate $T_n = h\nu_n = h\omega_n/2$, which, unfortunately, was not what he wanted. How could the factor n needed for equation 5 be pushed in? Bohr proposed two possibilities in contradiction with one another and with the usual formulations of Planck's theory. One possibility was that during its capture, the electron emitted n quanta each of frequency $\omega_n/2$. The other was that only one quantum of frequency $n\omega_n/2$ came

time to work Nicholson out of his system; McCormmach, Arch. hist. exact sci. **3**, 180–1 (1966); *CW* **2**, 270, 315.

out. The multi-quantum condition seemed unlikely because, as Bohr observed, the electron might be expected rather to change its frequency as it lost energy. The second formulation, one quantum of frequency $n\omega_n/2$, evidently conflicted with the ad hoc argument that ν_n should be $\omega_n/2$.[130] In both pictures, the connection between the frequencies of the Balmer lines and the orbital motions of the electrons that somehow produced them was loose and opaque. For, from equation (5),

$$\nu_n = \omega_2 - n\omega_n/2. \tag{7}$$

The pertinence of Rayleigh's observation, that the frequency received by the eye might not be the frequency of motion of the radiating electron, was thus wondrously if perplexingly confirmed.

The third grounding avoided the analogy to Planck's theory in favor of an adroit limiting process. In a Balmer-like transition between neighboring orbits a long way from the nucleus ($n \gg 1$), the radiated frequency is

$$\nu_{n,n-1} = R[1/(n-1)^2 - 1/n^2] \approx 2R/n^3. \tag{8}$$

From the second equation (3),

$$\omega_n \approx \omega_{n-1} = 4\pi m Z^2 e^4/n^3 h^3. \tag{9}$$

Setting equations (8) and (9) equal, which amounts to requiring that in the stated limit the frequency of light radiated in accordance with Bohr's quantum theory equals the frequency of the radiation as computed by classical physics, Bohr recovered equation (6). But we know that if equation (6) holds and ordinary physics reigns in the stationary states, equation (5) follows.[131]

The fourth grounding is the familiar quantization of the angular momentum. It may seem to be a reworking of equation (5),

$$T/\omega = \pi G = nh/2, \quad G = nh/2\pi, \tag{10}$$

but the two formulations differ in physical meaning. Equation (5) is a condition on the radiation emitted by an electron falling from "infinity" into the nth stationary state: it releases either one quantum of frequency $n\omega_n/2$ or n quanta of frequency $\omega_n/2$ at the cost of an energy (in either case) $T_n = nh\omega_n/2$. Equation (10) is a condition on the stationary states and involves only dynamical quantities characterizing them. Bohr preferred this condition when discussing atoms other than hydrogen and ionized helium, and set out as the universal condition of the ground state that every electron in it had exactly one quantum of angular momentum $h/2\pi$. He discarded this condition in a year or so to admit electrons with higher values of G in the ground states.[132] By then he had jettisoned the first two derivations, the analogies to Planck's radiation theory, as "misleading." The deeper third derivation he maintained as the primary illustration of the "Correspondence Principle," a method he developed between 1913 and 1919 for advancing the non-classical theory of the quantum atom by reference to classical calculations at loosely defined limits.[133]

[130] PM **26** (Jul 1913), 4-5, 7-8, in *CW* **2**, 164-5, 167-8.

[131] It might appear that the requirement that $\nu_{n,n-1} \approx \omega_n \approx \omega_{n-1}$ conflicts with equation (5), from which, if $\omega_n \approx \omega_{n-1}, \nu_{n,n-1}$ would be $\omega_n/2$. However, $\nu_{n,n-1} = [n\omega_n - (n-1)\omega_{n-1}]/2 \approx [nd\omega_n/dn - \omega_n]/2 = \omega_n$ by equation (3).

[132] Heilbron, *Moseley*, 102–5 (1974), and Isis **58**, 451–70 (1967); Bohr, PM **26**, 24–5 (1913) (*CW* **2**, 184–5), and *CW* **2**, 385.

[133] *CW* **2**, 294–6 (text of Dec 1913).

In stating the condition on the angular momentum, Bohr stressed that, although there could be "no question of a mechanical foundation of the calculations," they lent themselves to "a very simple interpretation... by help of symbols taken from the ordinary mechanics."[134] He had not entered a similar caveat when introducing the fundamental relation $(T/\omega)_n = nh/2$, probably because it had the authority of Planck's theory of radiation, on which he claimed to have founded his theory "entirely."[135] He soon found it necessary and advantageous to interpret his symbols literally. He did so to defeat a challenge from spectroscopists who objected that the theory had no place for a series they ascribed to hydrogen because it satisfied the Balmer-like rule,

$$\nu_n = R^*[1/(3/2)^2 - 1/(n/2)^2]. \tag{11}$$

Bohr agreed that he had no place for it. He rewrote equation (11) as

$$\nu_n = 4R^*[1/3^2 - 1/n^2], \tag{12}$$

and, invoking equation (9) with $Z = 2$, ascribed the series to ionized helium. To this ingenious solution the spectroscopists objected that R^* did not quite equal the Balmer $R = R_H$. Bohr parried this thrust by making the nucleus and its electron rotate around their common center of gravity, as ordinary mechanics required; and so replaced m in his equations with $m/(1+m/M)$, where M is the mass of the hydrogen nucleus. Then $4R^*/R_H = 4(1 + m/M)/(1 + m/4M) \approx 4 + 3m/M = 4.00162$ with $m/M = 1/1850$. The measured value was 4.0016. Soon spectroscopists detected the lines of the "Pickering-Fowler" series (equation 12) in helium carefully purified from hydrogen, and accepted Bohr's reassignment of the lines to helium.[136]

When Einstein heard about this confirmation from Hevesy, he said that Bohr had made a great discovery. He did not mean the identification of the Pickering-Fowler lines with helium, but the assumption, expressed in the consequence $\nu_n = \omega_2 - n\omega_n/2$ of the fundamental postulate $T_n = nh\omega_n/2$, that the radiated frequencies are not related transparently to the orbital frequencies of the electrons supposed to be involved in their production. The wording of Hevesy's report is instructive: "When he heard this [about the Pickering-Fowler series] he was extremely astonished and told me, 'Then the frequency of the light does not dep[e]nd at all on the frequency of the electron' (I understood him so??). 'And this is an enormous achie[v]ement.'"[137] Evidently, Einstein's specification of Bohr's "enormous achievement" as the finding that $\nu \neq \omega$ puzzled Hevesy. The fact that decoupling the frequencies deprived physicists of the tool with which they customarily approached radiation phenomena did not bother the chemist in him. But even to physicists who accepted Planck's quantum theory and the need to introduce some sort of discontinuity to obtain it, the loss of the connection between observed and theoretical quantities, between effect and cause, was distressing as well as disarming. The difficulty was disguised in Planck's theory, since, as in ordinary physics, it makes the frequency of the quantum oscillator that of the radiation it emits. But that was a consequence of the special model Planck used, the harmonic oscillator, which vibrates at the same frequency no matter what its energy.

Rutherford's initial reaction to the quantum jump perfectly expressed the malaise it stimulated. "[T]he mixture of Planck's ideas with the old mechanics makes

[134]Bohr, PM **26**, 15 (Jul 13), in *CW* **2**, 175.
[135]Bohr, PM **26**, 25 (Jul 13), in *CW* **2**, 185.
[136]Bohr, Nature **92** (23 Oct 13), in *CW* **2**, 275.
[137]Hevesy to Bohr, in *CW* **2**, 532 (23 Sep 13).

it very difficult to form a physical picture of what is the basis of it. There appears to me one great difficulty in your hypothesis, which I have no doubt you fully realize, namely, how does an electron decide what frequency it is going to vibrate at when it passes from one stationary state to the other? It seems to me that you would have to assume that the electron knows beforehand where it is going to stop."[138] Strict causality, implied by Rutherford's jocular attribution of foreknowledge to the electron, and reliance on mechanical modeling, expressed by his reference to its vibration during transition, would be painful to abandon.

By pinpointing as Bohr's great contribution an abrogation of ordinary physics marvelously and mysteriously confirmed (in the case of the helium spectrum) by requiring that it apply in detail, highlighted the wonderful ambiguity of Bohr's creative thought. Two aspects of this creativity struck Einstein. One was courage. Einstein told Hevesy that he had had similar ideas once "but had no pluck to develop [them]."[139] Bohr often described himself in his letters to Margrethe as a wild man. It took some courage to be bolder in physics than Einstein! Courage, yes, and intelligence, but also tact. This is the second quality that Einstein remarked in Bohr: his unfailing scientific tact. "I have complete confidence in his ways of thinking." Bohr could keep a steady eye on the goal despite all the contradictions and ambiguities in his path. It was more than an Einstein could manage: "if I did not have so many diversions, quantum problems would long since have driven me to the mad house."[140]

3 Reflections

Reprise

Bohr would not have created his quantum atom if he had followed the standard course of Danish scientists and finished his training in Germany. Except for his brother-in-law Niels Erik Nørlund, who stayed in Copenhagen, the other scientist members of the Ekliptika went to Göttingen (Harald Bohr, Edgar Rubin), thus copying their colleagues Niels Bjerrum (physical chemistry, Berlin) and Hans Hansen (spectroscopy, Göttingen). Why did Bohr choose England? Professionally, he went with the expectation of working with Thomson, although, with his interests, Lorentz in Leyden might have been a better choice. Thomson was more alluring, however, prolific in ideas, clever in mathematics, and playful in physics, and the Cavendish Laboratory under his direction was a leading and lively center. Moreover, Cambridge had a few other mathematicians and mathematical physicists worth listening to, Larmor and Jeans, for example, and Harald Bohr's colleague Hardy. To these attractions must be added, and perhaps given first place, Bohr's admiration of British culture.

Although we cannot quite say of Bohr, as the professor of classical philology at the University of Copenhagen, Anders Bjørn Drachmann, said of himself, "what I have I have drawn from Carlyle and Kierkegaard,"[141] Bohr had read almost as widely and deeply in English as in Danish literature. No doubt his grandmother, whom he knew well, gave him first-hand information about the country in which she had grown

[138]Rutherford to Bohr, in *CW* **2**, 583 (20 Mar 13).
[139]Hevesy to Bohr, in *CW* **2**, 532 (23 Sep 13).
[140]Einstein to Paul Ehrenfest, in Einstein, *Papers* **13**, 202–3, 188 (22 and 15 Mar 22).
[141]Drachmann to Jacobsen, 24 Jul 14, in Jacobsen and Brønsted, *Relig. brevv.*, 154 (1964).

up, as did visiting English relatives, who would offer welcome and support should he return the visit.[142] He set a high value on family ties. Another sort of family connection was available to him in England in the persons of his father's former students who had risen to professorial posts there. "Believe me [he wrote Harald], it is nice that one's name is known." One of those who knew the name, James Lorrain Smith, Professor of Physiology at the University of Manchester, introduced Bohr to Rutherford. Niels was more at home with members of this academic quasi-family than with his fellow students at Cambridge. During his visit to Lorrain Smith in Manchester he felt (as he wrote his mother) "how exceedingly wonderful it is for me to be among real friends again!"[143]

I have mentioned that the Bohr family knew the novels of Charles Dickens so well that Ellen Bohr could liken Margrethe to half a dozen heroines from *Great expectations* to *Little Dorritt*. A letter from Niels to Harald contains an apt reference to a minor character from *Our mutual friend* to express his unease at English dinner parties, and, as we know, Niels turned to *David Copperfield* in his isolation in Cambridge to pass the time and polish his English. Dickens' exaggerations appealed to Bohr's turn of mind; for caricature is a way of bringing out a significant trait, or partial truth, in a memorable and suggestive way. Something similar may be said about Thomson's style in physics. He employed different models to bring out different aspects of the phenomena he studied and to give variable depths to his theories: thus Faraday's tubes, the vortex atom, electromagnetic mass, but also electrons conceived as charged billiard balls and atoms like plum puddings with moving raisins, or like piles of dipoles, held together by forces known and unknown to electromagnetism. "Unbelievably full of ideas," he had a capacity to dream up analogies, caricatures, possibilities, and far-fetched connections of Dickensian dimensions.[144]

Bohr's groundings of his quantum atom may also be considered as apt caricatures. The first two, which rested on analogies to Planck's oscillators, portray the partial truth of Planck's quantum theory of radiation. The third one, which declared an asymptotic agreement between calculations based on the quantum atom and on ordinary radiation theory, portrayed the mutual limitations of ordinary and quantum physics. The fourth one, the condition on the angular momentum, conveyed the partial truth that the application of ordinary physical ideas to the microworld is entirely symbolic.

So much at present for what, following Professor Drachmann's division of inspiration, may be called in a Pickwickian sense the English items in Bohr's mental furniture. I say Pickwickian because Bohr did not share Drachmann's admiration for Carlyle's *On heroes and hero worship*, which Margrethe sent him that he might see himself as she saw him. Although he liked the first chapter, on the man-god Odin ("when I see the briefest reference to the Old Nordic countries then my heart flares up so wildly, so wildly"), he decided that Carlyle wrote more as a sermonizer than as a philosopher. Margrethe was hurt by this rejection of her idea of choice literature and remembered the incident for the rest of her life. The episode had the value, however, of calling forth as an apology Bohr's version of Høffding's confession of faith quoted earlier: there exist many sorts of truth, everything of value is true, belief in multiple layered truths can substitute for religion.[145]

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[142] AH, 36, 50, 67, 72, 74, 152.
[143] Quotes from, resp., Niels to Harald, in *CW* **1**, 519 (29 Sep 11), and to Ellen Bohr, in AH, 34 (4 Nov 11).
[144] Bohr to Oseen, in *CW* **1**, 427 (1 Dec 11).
[145] Niels to Margrethe, AH 157, 51–2 (17 Dec 11 (quote), and 15 Jan 12).

Turning now to Bohr's Danish side, no psychoanalytic penetration is needed to perceive that a student of Høffding's could contemplate with the utmost satisfaction the crisp dichotomy between stationary states, in which electrons behaved as if Newton had designed the atom, and quantum jumps, in which not even a Newton could follow them. Høffding had no problem assimilating Bohr's theory into his epistemology.[146] Had he not taught that the world consisted of the continuous, describable, and rationally explicable, and the discontinuous, irrational, and novel? Here, in a simple case, Bohr had found one of those closed doors at which continuity must stop and a jump be made; or, to adopt the words Niels used to reconcile his future mother-in-law to her daughter's marriage outside the church, a place where we confront the demonstrable truth that there are things human beings cannot understand. What struck Rutherford and Einstein as spectacularly bold conclusions were for Bohr only what was to be expected, indeed, what was to be sought. "The world is not complete, not harmonious, not rational; therefore there is work to be done." That was the world according to Høffding, the good old pluralist and irrationalist, the Copenhagener Geist, who could instill his message "without the receiving person noticing it."[147]

The quantum atom presented two impasses in addition to the brick wall between stationary states and sudden jumps, where, as James expressed the predicament, thought fails to continue from above and from below. Following his thought as far as it continued downwards, that is, to the atomic nucleus, Bohr locked up radioactivity in the unexplorable region where alpha and beta particles originated spontaneously, by chance, independently of the physical and chemical forces acting on the atom, that is, irrationally. The blockage from above came in the ethereal spaces into which the jumping electrons sent their rays, where an irremediable discontinuity loomed between the production and transmission of radiation; for although Bohr referred to Einstein's theory of the photo-effect in discussing absorption by his quantum atom, he could not accept the associated concept of light corpuscles, and again shelved the problem.[148] In both directions, the nuclear atom allowed him to delimit his domain and postpone consideration of phenomena whose analysis might well require the invention of more principles unknown to ordinary physics.

The high tolerance for ambiguity that distinguishes Bohr's thought is a trait often developed by people assimilated into one culture who maintain ties to another. As we know, during 1912 Bohr had several strong reminders, if he required any, of the tensions and ambiguities of assimilation. He spent time with his Jewish relatives in Britain, deliberated over the ethical and social consequences of his refusal to marry in church, witnessed the first success of Nathansen's popular play about intermarriage, and, perhaps, heard or heard of Brandes' lectures on the Jewish spirit in Danish culture. Bohr displayed many traits reckoned as Jewish by Nathansen and Brandes: boldness, assertiveness, irony, constant striving, addiction to thinking, openness to ideas, closeness to family, humanism, and, peculiarly, talkativeness. Here Bohr qualified unquestionably; he was forever quoting, himself and others, and he developed his papers in discussion with his assistants before he dictated them. He also over-qualified in feeling guilt, a commonly alleged characteristic, which however, Nathansen and Brandes do not mention.

Did the creative tensions of assimilation or its allowance for ambiguity play a

[146] Høffding to Bohr, in *CW* **10**, 511–14 (20 Sep, and reply, 22 Sep 22); Bohr, ibid. 322 (text of 1931).
[147] Quotes from, resp., Høffding, Jl phil. psych. sci. methods **2**, 92 (1905), and Bohr, *CW* **1**, 321.
[148] Bohr, PM **26**, 16–17 (1913), in *CW* **2**, 166–7.

decisive part in Bohr's creativity? Did the epistemology he derived from Høffding and Kierkegaard? Did the emphasis on the life of the mind characteristic of high Jewish culture and refined romantic Danish intellectuals like Kierkegaard and Høffding? Did the unstinting support of his family? It would not be safe to rule any of them out.

Creativity

The analogy between the creative act and a quantum jump is supported by Poincaré's famous account of his first important discovery. The incident dates from 1880, the account, from a lecture to the Parisian Société de psychologie, from 1908. For two weeks before the discovery, Poincaré had strained to prove the false theorem that a certain mathematical species could not exist. On the eve of the breakthrough he drank a cup of coffee, could not sleep, and watched as "ideas surged up in a crowd... bumped against one another... hooked on to one another." In the morning he realized that the questionable species existed and wrote up the proof in two hours. He then went on an excursion. Suddenly, as he stepped onto the bus, an important property of the new species darted into his head although he had not been thinking about mathematics. "[T]he idea came to me, without anything in my previous thoughts having prepared me for it."[149]

From this and similar stories by the mathematician Jacques Hadamard, who echoed Poincaré's experience and gave many other instances from the arts and sciences, psychologists have worked out that the creative act occurs when, after intense pondering, the creator *in potentia* relaxes and lets his or her mind roam freely and unconsciously until, spontaneously, it connects disparate things together and thrusts a new idea into consciousness.[150] If so, we might suppose that the greater the fund of knowledge and experience brought to bear, the wider the spontaneous connections will be, and the greater or crazier the discovery. Thus Dugald Stewart, the great exponent of Scottish Common Sense Philosophy, wrote commonsensically that "all the materials with which experience and reflection have supplied us" figure in the creative act.[151] The creator explores, winnows, plays with, rearranges, selects, connects elements in his rich unconscious or subconscious mind, and, by luck or accident, in a blind Darwinian moment, fishes up a great novelty.[152]

To William James, "the highest order of mind" works by "cross-cuts and transitions from one idea to another, [by] the most rarefied abstractions, the most unheard-of combination of elements, the subtlest associations of analogy... everything is fizzling and bubbling about in a state of bewildering activity, where partnerships can be joined or loosened in an instant." "Our consciousness works itself out of a dark chaos," says Høffding, "and its sporadic elements are combined through an involuntary synthetical process." Creativity implies a discontinuity in this involuntary process, for, "more than anything else, [it] releases locked powers, and opens up the greatest tasks in the realm of life no less than in the realm of science."[153] Even the creator does not know how anything useful erupts from this depth. Mozart: "When I cannot sleep, thoughts crowd into my mind.... Whence and how do they come? I

[149] Quoted in Gray, *Poincaré*, 216–17 (1912), from Poincaré, *Science and method*, 51–2 (1914).
[150] Hadamard, *Essay*, 12–14 (1945) (Poincaré), 15–16 (Helmholtz, Langevin, Ostwald).
[151] Stewart, *Elements*, 323 (1802).
[152] Simonton, *Origins* (1999), chapt. 2, sets out the analogy between creativity and cut-and-try Darwinian selection.
[153] Høffding, in Feuer, *Einstein*, 115–16 (1974), the second quote coming from Høffding, *Problems* (1905), 8.

do not know and I have nothing to do with it."[154] A certain problem had stumped the great Gauss for two years. Then, suddenly, the answer came, "not on account of my painful efforts, but by the grace of God."[155] Here we may truly say that our continuous thought has struck an impasse, or irrationality, from above.

With respect to lesser creators than God, the heroic character of their genius, as well as the radical contingency and spontaneity of invention, would seem to block further inquiry here below. We are informed by Michael Faraday, in words that apply well to himself, "Every great man of the first rank is unique. Each has his own office in the historical procession of sages. That office did not exist even in the imagination, till he came to fill it, and none can succeed to his place when he has passed away."[156] More obscure yet, the mind of the creator might be that of a poet. Faraday resorted to the "neat and clean power of poetry, the mistress of all discovery," in developing his ideas. "You can scarcely imagine how I am struggling to exert my poetical ideas just now for the discovery of analogies and remote figures...for I think that is the true way (corrected by judgment) to work out a discovery."[157]

Planck taught that the great theorist is a great artist, indeed, a romantic, working "not only for momentary success but for eternity;" he praised colleagues for their artistic, powerful, groping imaginations and received in return Einstein's admiration for his "truly artistic style" and "artistic compulsion."[158] Bohr liked poetry and as a boy learned large swatches of Goethe and Ibsen, which he could recite with greater clarity and emphasis than he could articulate his scientific ideas. He remained a great reader. A colleague encountered him returning from a walking tour in Norway carrying a thick backpack; "he had naturally taken a little library with him to study on the way."[159] And he saw in himself the "boiling blood," the boldness of conception, "the wandering thoughts and wild dreams," associated with the romantic poet and the Icelandic bard.[160] You do not have to compose verses to be a poet; likely stories, world pictures, verisimilitude will do. "Hence, he is call'd a Poet, not hee which writeth in measure only; but that fayneth and formeth a fable, and writes things like the Truth."[161] "[W]here metaphysical hypotheses live and move...thought and poetry are often insensibly blended." Thus Høffding, who, approaching his subject as usual from above and below, makes poetry the only route to the highest truths and also to ordinary observation, "which sometimes supposes the talent of the poet, sometimes of the experimentalist."[162]

This last image suggests a way around the opaque operations of the creating mind. Elegiac poetry is dead. No one now writes about great deeds in 20,000 heroic couplets. The art of portraiture no longer coaxes forth Titians or Rembrandts. Grand historical painting is a thing of the past. In short, art forms have their fads, flower, and decay. It is the same with physics. The style that originated with Stokes and Kelvin, whose mechanical models "stirred their souls like the memories of child-

[154]Mozart, quoted in Hadamard, *Essay*, 16 (1945).
[155]James, *Great men*, 456 (1880), quoted from Simonton, *Origins*, 28–9, 44 (1999); Gauss, quoted in Hadamard, *Essay*, 15 (1945).
[156]Quoted by Maxwell, in *Papers* **1**, 358 (1890).
[157]Respectively, Ben Jonson, *News*, in *Works*, 435 (2012), and Faraday to Schönbein, 13 Nov 1845, in Kahlbaum and Derbyshire, *Letters*, 149 (1899).
[158]Heilbron, *Planck*, 52 (2000).
[159]Niels Møller to J.P. Jacobsen, 9 Jan 18, in Jacobsen and Brønsted, *Relig. brevv.*, 262 (1964).
[160]AH, 84 (second quote); boiling blood, passim, e.g., 23 Apr and 27 May 12.
[161]Jonson, cited by Gordon, Jl Warb. Court. Institute **12**, 158–9 (1959).
[162]Høffding, Int. jl ethics **22**:2, 143 (quote), 151, 141 (quote) (1902).

hood," passed through Maxwell and Rayleigh to end in Thomson.[163] The relatively modest means and first approximations with which Rutherford achieved his extraordinary results would not take him far today. The conception of their field and their responsibility to fit physics into a coherent and wider worldview, which characterized the romantic physicists from Planck to Bohr, have not been mainstream for decades. History moves through stages, says Høffding, echoing Comte, Hegel, and Kierkegaard, and "a new truth is necessary, when a new stage in life is reached."[164]

The way forward then is to try to match mental states with the state of science, to identify the powers and preparation apt for the "search for the new forms of thought," and for the upwelling of the metaphors, on which creativity often turns.[165] What qualities of mind have the problems and content of physics, and its status and image in society, attracted from time to time? How did the wider society prepare people with these qualities? Is it credible that a man of Bohr's mental makeup, if any such could be trained today, would become a physicist? Rather than trying to trace creative acts, the historian might ask more fruitfully how the creation fits with the pre-existing contents of the creator's mind, and try to describe how the main furnishings got there. The outcome of the exercise may be a better understanding of the scientific enterprise as well as of individual investigators.

The inquiry necessarily would go deeper than the division of scientists into Classical and Romantic types, as in Wilhelm Ostwald's *Grosse Männer*, or into groups with different guiding "themata," as in Gerald Holton's analyses of the scientific imagination. For although (to take Holton's themata) unity, economy, symmetry, conservation, continuity, discontinuity, and so on can describe the gist of a scientist's work, as Holton shows in his sketches of Einstein, Bohr, Poincaré, and Kepler, they operate rather as retrospective categorizations of the physics produced than as inventories of the furnishings of a creative mind.[166] "We live forwards and reason backwards."

Bohr once tried to capture the characteristics of the Danish mind. He decided on "the immediate combination of an openness to the lesson brought to us from the outside or that we bring home ourselves, and an adherence to our outlook on life, determined by our inheritance and destiny." He instanced Møller, "the most Danish of all Danish writers," and Kierkegaard, perhaps the most astute and profound, especially in the last pages of *Stages*.[167] He did not attempt to apply a similar analysis to himself when asked in an interview to throw light on his own creative style as represented in the first paper of his trilogy. Rather than appealing to the unconscious generation of great ideas, or suggesting connections with the qualities he had identified in the Danish mind, he dismissed the telltale ambiguities and contradictions of his formulations as absurdities. He could not have intended those curiosities seriously, he said, although later in the same interview he recalled defending every word of the final draft of his paper as "quite essential to the argument" when Rutherford offered to cut it down.[168]

What then was the justification of the averaging to get the factor 1/2 in the fundamental equation $T_n = nh\omega_n/2$? "That was just the stupidity of the way of

[163] Maxwell, "Address," in *Papers* **2**, 220, quote (text of 1870).
[164] Høffding, Int. jl ethics **22**:2, 139, 147 (quote) (1902).
[165] Maxwell, "Address," in *Papers* **2**, 227 (text of 1870).
[166] Holton, in *Mélanges* **2**, 261–4 (1964), *Science*, 95–8, 106 (1965), Daedalus, 1970, 1030–3, and *Scientific imagination*, 13–18 (1998).
[167] *CW* **10**, 265 (text of 1940).
[168] Bohr, *Interview*, 61 (1962).

looking at it." What about the odd analogies to Planck's theory? "That is taken too seriously, you see. It's not so, actually. . . .It was not taken seriously at all. There are some sentences about this which I actually agree are nonsense. . . .It is hard for me to see what it means." What about the condition on the angular momentum? "It really would have been much more beautiful if it had all been left out." And the entire approach? "Most of it is sheer nonsense."[169] Sheer nonsense in a paper that won the immediate applause of Einstein? Sheer nonsense as the considered product of a mind richly furnished with choice literature in three languages, original solutions to epistemological problems, the intellectualized debris of a religious crisis, the liberal culture of the assimilated Jew, the tact to advance through paralyzing ambiguities, and, of course, a deep understanding of physics?

I have tried to show that the parts of the trilogy that Bohr judged to be nonsense fifty years after he wrote them help us to do what he could not do, even in retrospect: think about the style of his thinking, evaluate what in his mental makeup was pertinent to his habit of thought. Scientists who now know the "right" answer may find it particularly difficult to recover the mental forces they mobilized to make their necessarily imperfect creation. Whatever the value of the present exercise, it has led through a literary and cultural landscape worth exploring and has defined a psychological problem, which, if not solvable, is worth attempting. It may be, as Høffding said, that we create by "an involuntary synthetical process" or by "an involuntary symbolizing of unconscious tendencies and dispositions." Nonetheless, though we may never devise a satisfactory causal account of scientific creativity, we need not therefore stop with the famous physician and bookman John Shaw Billings, who, as President of the Philosophical Society of Washington, explained the "so-called thirst for knowledge" of men of science as something like an instinctive desire, "such as that which leads a rat to gnaw."[170]

[169]Ibid., 8, 57–9.
[170]Høffding, Int. jl ethics **22**:2, 138, 139 (1902), and Billings, Science **8**, 544 (1886).

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