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Time and Relativity

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1 Introduction

Time has many facets, related to most fields of human endeavour, and to many separate fields of science :

- Metaphysics : e.g. Heraclitus' *"Panta rei"* ("everything flows") ; Zeno's arrow, Leibniz's relational time, Kant's ideality of time,. . .
- Spirituality : samsara, maya, sunyata, brahmanda (cosmic egg), orphism, bereishit (book of genesis), eternal return through ekpyrosis, death and resurrection, eternity,. . .
- Psychology : awareness, flow of consciousness,. . .
- Literature : e.g. from Virgil's "fugit irreparabile tempus" to Proust's *In Search of Lost Time*.
- Music : rhythm, tempo, frequency,. . .
- Historical studies : from Herodotus to Fernand Braudel.
- Technology : from Sun dials to LED watches.
- Biology : circadian cycles, aging, programmed cell-death, evolution of species, mitochondrial DNA mutation rate,. . .
- Sociology : working hours, summer time,. . .
- Probability Theory : Bayesian inference, stochastic differential equations, Markov processes, Kolmogorov-Chaitin complexity,. . .
- Astronomy : day, month, year, celestial mechanics, the origin of the solar system, chaos,. . .
- Metrology : atomic clocks, lasers, frequency comparisons,. . .
- Thermodynamics : irreversibility, the Second Law ($dS/dt \geq 0$),. . .
- Statistical Physics : Boltzmann's equation, Boltzmann's H-theorem, fluctuation-dissipation, Onsager's relations,. . .
- Chemistry : chaotic chemical reactions, Belousov-Zhabotinsky, self-organisation,. . .
- Hydrodynamics : Navier-Stokes, viscosity,. . .
- Information Theory : from Brillouin and Szilard to Shannon, Landauer and Bennett.

- Electromagnetism : retarded potentials versus advanced ones, radiation, the Einstein-Ritz debate, Wheeler-Feynman,. . .
- Classical Dynamics : Liouville's theorem, periodic systems, quasi-periodic motions, Poincaré recurrences, Lyapunov exponents, chaos, strange attractors,. . .
- Geology : . . .
- Paleontology : . . .
- Archeology : . . .
- Special Relativity : time dilation, twin paradox, light-cone, Poincaré-Minkowski spacetime geometry,. . .
- General Relativity : gravitational redshift, GPS, warped spacetime, black holes, worm-holes, time travel, closed time-like curves,. . .
- Astrophysics : Doppler effect, gravitational redshift, pulsar timing,. . .
- Cosmology : big bang, expansion of the universe, big crunch, spacelike singularities, inflation, eternal inflation,. . .
- Quantum Theory : "collapse of the wave function", the measurement issue, the time-energy uncertainty relation, the Zeno effect,. . .
- Nuclear Physics : nuclear decay, radioactive isotope dating,. . .
- Atomic Physics : stationary states, quantum transitions, lifetime of unstable states, Ramsey transitions,. . .
- Quantum Field Theory : Stückelberg-Feynman propagators, Wick rotation, CPT,. . .
- Quantum Gravity : spacetime foam, (de-)emergence of space time at spacelike singularities, gauge-gravity duality, holography,. . .

This (certainly incomplete) list illustrates the all pervading significance of the concept of Time. The present contribution will focus only on a few aspects of Time, namely those relating its apparent fundamental irreversibility ("Second Law") to Einstein's revolutionary ideas about Space, Time and Matter, and their import in current developments in physics and cosmology. Before coming to grips with these issues, we shall set the stage by recalling the "common conception of Time" (which was enshrined in Newton's *Principia*), as well as the ground-breaking ideas introduced by Boltzmann in 1897. Our treatment will be rather brief and superficial. The interested reader is referred to the books of Paul Davies [1], Brian Green [2], Alex Vilenkin [3] and Sean Carroll [4] for more complete discussions, and references to the huge literature on Time.

2 The common conception of Time

In an often quoted sentence of his *Confessions*, Saint Augustine wrote : "What is time ? If no one asks me, I know. If I wish to explain it to one that asketh, I know not." However, when pressed to answer the question "what is time ?", it seems likely that the most common answer would roughly be that Time is something exterior to the material universe around us, that "passes", or "flows", thereby creating our perception of reality as a "now", as well as inexorably dragging this perception from the past to the future. I do not know for how long this conception of time has been commonly held by human beings (nor do I know whether other animals share it). Among the ancient Greeks, it seems that, with the important exception of Parmenides and his school, the Heraclitean view of a "flow of time" (common to us and the universe) was considered as the standard one. Jumping to more recent times, it seems that mechanical clocks appeared in European convents near the end of the thirteenth century. [They were used to indicate the passing of time to the monks, whose daily prayer and work schedules had to be strictly regulated.]

Later, mechanical clocks became part of the everyday life of ordinary citizens, through the construction of clock towers, notably on cathedrals. Their presence in the city contributed to imposing the conception of a universal time, before the basic scientific advances of the seventeenth century. We have in mind here Galileo (who noticed the isochronism of small pendulum oscillations, and introduced time in the dynamical description of reality), Huyghens (isochronism of a cycloidal pendulum), and Newton. Let us recall how Newton describes his conceptions of time in the scholium to the Definitions at the beginning of his monumental Philosophiae Naturalis Principia Mathematica (1687) [5] :

"Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place, and motion, as being well known to all. Only I must observe, that the common people conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration : relative, apparent, and common time is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time ; such as an hour, a month, a year.

II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces, which our senses determine by its position to bodies ; [...]".

The "Newtonian" concept of absolute time was developed at a time when long-case clocks were used in private homes, and when people started to carry pocket watches. [It seems that Blaise Pascal (1623-1662) was the first to attach his pocket watch to his wrist.] Since that time the conjunction of the tick-tock of public or individual clocks and watches, and of the successful development of the Newtonian description of reality (from celestial mechanics to industrial devices) has "hammered" the common conception of time, recalled above, deeply into the minds of most people.

3 Boltzmann and the first time revolution

The common (and Newtonian) concept of time underwent a first revolution at the end of the nineteenth century, through the work of Boltzmann on the second principle of thermodynamics.

Let us recall that the Second Law of thermodynamics states that the entropy, S, of an isolated system can only increase with time : $dS/dt \geq 0$. This law formalizes, in particular, the many irreversibilities that one observes everyday. E.g. the fact that we see ice cubes melting in a glass of hot water, but we never see a glass of tepid water separating into ice cubes and hot water. Clearly, the Second Law also underlies the fact that we have memories of the past but not of the future, essentially because one needs to have at hand low-entropy reservoirs either to record information on "blank slates" or to "erase" already recorded information (as was discussed by Brillouin, Szilard, Landauer and Bennet ; see references in [4]).

We recall that Boltzmann thought he had succeeded (in 1872) in deriving the irreversible increase of the entropy of an isolated mechanical system, $dS/dt \geq 0$, from an innocent-looking assumption about the number of collisions in a gas (the so-called "Stosszahlansatz"). [See [6], p. 88.] However, several scientists raised objections to the "proof" of the "H-theorem" of Boltzmann. [We recall that Boltzmann discussed the evolution of the quantity $H \equiv \int f \ln f \, dq \, dp$ which is the *negative* of the (Boltzmann) entropy S a gas, described by its one-particle phase-space distribution $f(q,p)$.] First, soon after Boltzmann published his "theorem", Lord Kelvin, Maxwell, Loschmidt and others pointed out that the time-symmetry of the underlying (Newtonian) dynamics of colliding atoms made it impossible to derive, as a mathematical theorem, a time-dissymmetric result such as $dS/dt \geq 0$. This led Boltzmann to stating that the Second Law had only a *statistical* validity, though an overwhelmingly probable one. Twenty years later ($\sim$ 1896), Zermelo raised a new objection based on the recurrence theorem of Poincaré (1890). Indeed, the fact that an isolated system having a *compact* (or, at least, *finite measure*) phase space will recur infinitely many times to a state very close to its initial state (however "improbable" it may be) seems to undermine the existence of a molecular basis of the Second Law. This objection (or, more precisely, a second, related objection of Zermelo concerning the choice of initial state) led Boltzmann to proposing radically new ways of thinking about the physical origin of the Second Law. First, Boltzmann acknowledges that the only way to explain, through considerations of the (reversible) dynamics of molecules, the deeply irreversible nature of the Second Law, is by means of an *assumption* about the state of the universe. He calls this assumption, *assumption A*, and introduces it as follows (Boltzmann 1897, see p. 238 in [6]) :

"The second law will be explained mechanically by means of assumption A (which is of course unprovable) that the universe, considered as a mechanical system – or at least a very large part of it which surrounds us – started from a very improbable state, and is still in an improbable state."

Actually, this rather convoluted sentence means that Boltzmann is here hesitating between two different assumptions. Later in his text, he will clarify their differences, and refer to them as "two kinds of pictures". For clarity, let us give them two different names, say :

- assumption $A^{\rm global}$: the *entire* universe started from a very improbable state, and is still in an improbable state ;
- assumption $A^{\rm local}$: only the (large but) *local patch* of the universe that surrounds us finds itself at present in a very improbable state.

The meaning of assumption $A^{\rm global}$ is clear. On the other hand, to understand the meaning of assumption $A^{\rm local}$, it is worth to quote the sentences in which Boltzmann explains the second possible "picture" for understanding the origin of the Second Law :

“However, one may suppose that the eons during which this [1] improbable state lasts, and the distance from here to Sirius, are minute compared to the age and size of the universe. There must then be in the universe, which is in thermal equilibrium as a whole and therefore dead, here and there relatively small regions of the size of our galaxy (which we call worlds), which during the relatively short time of eons deviate significantly from thermal equilibrium. Among these worlds the state probability increases as often as it decreases. For the universe as a whole the two directions of time are indistinguishable, just as in space there is no up and down. However, just as at a certain place on the earth’s surface we can call “down” the direction towards the centre of the earth, so a living being that finds itself in such a world at a certain period of time can define the time direction as going from the less probable to more probable states (the former will be the “past” and the latter the “future”) and by virtue of this definition he will find that this small region, isolated from the rest of the universe, is “initially” always in an improbable state. This viewpoint seems to me to be the only way in which one can understand the validity of the second law and the heat death of each individual world without invoking an unidirectional change of the entire universe from a definite initial state to a final state. The objection that it is uneconomical and hence senseless to imagine such a large part of the universe as being dead in order to explain why a small part is living – this objection I consider invalid. I remember only too well a person who absolutely refused to believe that the sun could be 20 million miles from the earth, on the grounds that it is inconceivable that there could be so much space filled only with aether and so little with life.”

The visionary nature of this remarkable text has only been appreciated rather recently, especially in the context of modern cosmology. In modern parlance, Boltzmann’s vision consists of :

- viewing our visible universe as a spatially and temporally localized entropy fluctuation within an infinite (or much larger) universe ;
- appealing (as emphasized in [7]) to a form of the Anthropic Principle : while, globally, the universe is in a “heat death” state, life can exist only in regions where a large enough fluctuation of the entropy away from its maximum takes place ;
- considering the “flow of time” as an *emergent illusory phenomenon*, locally induced by the local (spacetime) value of the entropy time-gradient.

I find the latter point the most revolutionary one from the conceptual point of view. It is not clear how many readers of Boltzmann fully realized that if indeed there exist, in the total spacetime [2], *antichronal regions* [3], i.e. local spacetime regions where sentient beings experience time as flowing in the opposite direction than us, this clearly means that the common conception of time as “flowing” externally to the entire universe is incorrect, and that the “flow of time” is a mere (biolo-psychological) illusion.

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[1] The text of Boltzmann is somewhat confusing as he is here grammatically referring to the “very improbable state” (of assumption A^{global}) in which “the entire universe finds itself at present”. We think, however, that he has in mind only a *local* version of assumption A.

[2] Let us emphasize that there is no real anachronism in phrasing Boltzmann’s view in terms of *spacetime*. Indeed, for instance, the influential book of H.G. Wells, *The Time Machine* (whose first chapter contains a vivid explanation of Time as a fourth dimension, additional to the three dimensions of Space) was published in 1895, i.e. two years before Boltzmann wrote his text.

[3] i.e. temporal analogues of *antipodal* regions on the Earth.

The "anthropic-fluctuation" scenario suggested by Boltzmann has been discussed, and rejected, by several physicists. Landau and Lifshitz, in the first English edition (1938) of their *Statistical Physics* volume [8] write :

> "Boltzmann attempted to remove this contradiction by his "fluctuation hypothesis". He suggested that in the relatively small part of the universe observed by us, chance fluctuations from the statistical equilibrium of the whole universe are taking place, or, in other words, the impression that the universe does not obey statistical laws is due to our part of the universe being in the course of an enormous fluctuation. The fact that it is possible to observe such a colossal fluctuation (over a volume exceeding 10^{75} c.c.) Boltzmann explained by supposing that just such a fluctuation is a necessary condition for the existence of the observer (a condition favouring biological development of organisms, for instance). This argument is, however, quite false, since there would be an enormously greater probability of a smaller fluctuation for which there existed for instance only a single observer, without the myriads of stars prepared for him, and in any case it would be sufficient for the possibility of observing the universe to have this deviation from equilibrium in a volume of only 10^{55} c.c. (containing the sun and nearest stars). In this connexion we should remark that the probability of fluctuations is so small that it is in general not possible to observe any appreciable fluctuations at all."

A similar argument was presented (in 1963) by Feynman in his *Lectures on Physics* [9] : "Therefore, from the hypothesis that the world is a fluctuation, all of the predictions are that if we look at a part of the world we have never seen before, we will find it mixed up, and not like the piece we looked at. If our order were due to a fluctuation, we would not expect order anywhere but where we have just noticed it." (because the probability of a fluctuation is proportional to $\exp(-S)$, so that "minimal" fluctuations corresponding to the minimal anthropic-compatible local decrease of entropy are a priori much more probable). Then, concerning the origin of the Second Law Feynman concludes that the "one-wayness" displayed by the evolution of any local (isolated) thermodynamical system "cannot be completely understood until the mystery of the beginnings of the history of the universe are reduced still further from speculation to scientific understanding."

On their side, Landau and Lifshitz offered, in the second English edition (1959) of [8], more precise suggestions about the ultimate origin of the Second Law. On the one hand, they point out that :

> "The answer is to be sought in the general theory of relativity. The point is that when we consider large regions of the system, the gravitational fields which they contain begin to become important. According to the general theory of relativity, the latter represent simply changes in the space time metric which is described by the metric tensor g_{ik}. In the study of the statistical properties of bodies, the metrical properties of space time can, in a certain sense, be regarded as the "external conditions" in which these bodies are situated. The assumption that after a long enough interval of time a closed system must eventually reach a state of equilibrium depends obviously on the external conditions remaining constant. But the metric tensor g_{ik} is, generally speaking, a function not only of the co-ordinates but of the time as well, so that the "external conditions" are by no means constant. It is important to note with this that the gravitational field cannot itself be counted as part of the closed system because in that case the conservation laws, which, as we have seen, are the very foundation of

statistics, would become simply identities. As a result of this, in the general theory of relativity the universe as a whole must be regarded not as a closed system, but as one which is in a variable gravitational field. In this case the application of the law of increase of entropy does not imply the necessity of statistical equilibrium."

On the other hand, they raise doubts about the possibility of deriving the Second Law from any (intrinsically time symmetric) classical theory, and they suggest that the time-dissymmetry present in Quantum Mechanics (when adopting the Copenhagen interpretation of measurements) might be related to the Second Law.

4 Einstein, Special Relativity and Time

Textbook presentations of Special Relativity often fail to convey the revolutionary nature, with respect to the "common conception of time", of the seminal paper of Einstein in June 1905. It is true that many of the equations, and mathematical considerations, of this paper were also contained[4] in a 1904 paper of Lorentz, and in two papers of Poincaré submitted in June and July 1905. It is also true that the central informational core of a physical theory is defined by its fundamental equations, and that for some theories (notably Quantum Mechanics) the fundamental equations were discovered before their physical interpretation. However, in the case of Special Relativity, the egregious merit of Einstein was, apart from his new mathematical results and his new physical predictions (notably about the comparison of the readings of clocks which have moved with respect to each other) the *conceptual* breakthrough that the rescaled "local time" variable t' of Lorentz was "purely and simply, the time", as experienced by a moving observer. This new conceptualization of time implied a deep upheaval of the common conception of time. Max Planck immediately realized this and said, later, that Einstein's breakthrough exceeded in audacity everything that had been accomplished so far in speculative science, and that the idea of non-Euclidean geometries was, by comparison, mere "child's play".

The paradigm of the special relativistic upheaval of the usual concept of time is the *twin paradox*. Let us emphasize that this striking example of time dilation proves that *time travel (towards the future) is possible*. As a gedanken experiment (if we neglect practicalities such as the technology needed for reaching velocities comparable to the velocity of light, the cost of the fuel and the capacity of the traveller to sustain high accelerations), it shows that a sentient being can jump, "within a minute" (of his experienced time) arbitrarily far in the future, say sixty million years ahead, and see, and be part of, what (will) happen then on Earth. This is a clear way of realizing that the future "already exists" (as we can experience it "in a minute"). No wonder that many people, attached to the usual idea of an external flow of time, refused to believe that the travelling twin will come back younger than his sedentary brother. This was notably the case of Bergson whose philosophy was based on a phenomenological intuition of time ("la durée"), experienced in its eternal flow as an "immediate datum of consciousness". Bergson characterized his view of time as follows [12] :

"Common sense believes in a unique time, the same for all beings and for all things

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[4] It is probable that Einstein knew neither the 1904 paper of Lorentz, nor the June 1905 short paper of Poincaré. For historical discussions and references to the original papers, see, e.g., the 2005 Poincaré seminar on Einstein [10] and the book [11].

[...]. Each of us feels themselves to experience duration [...] there is no reason, we think, that our duration is not as well the duration of all things."

Today, many experiments have confirmed the reality of time dilation (see the contribution of Christophe Salomon to this seminar). In spite of this, the special relativistic revolution in the concept of time has had little effect on "common-sense". In view of the fact that Copernicus' *De Revolutionibus* appeared in 1543, and that the new world view that this book pioneered started affecting "common sense" only a couple of centuries later, maybe we should not (yet) worry about the little effect that Einstein's 1905 insight has had on the man in the street.

5 General Relativity and Time

General Relativity opened the door to an even deeper upheaval of the common concept of time. However, most popular treatments of science have a tendency, when speaking of General Relativity (GR), and especially when describing relativistic cosmological models (Inflation, Big Bang,...), to use a language which suggests that GR reintroduces the notion of *temporal flow*, which Special Relativity had abolished. Far from it. The spacetime of GR is just a "timeless" as the special relativistic one. The Big Bang should not be referred to as the "birth" of the universe, or its "creation" *ex nihilo*, but as one of the possible "boundaries" of a strongly deformed (timeless) spacetime block.

Far from reintroducing the notion of temporal flow, the infinite variety of possible Einsteinian cosmological models furnish some striking examples of *conceivable "worlds"* where the unreality of this flow becomes palpable. For example, one can imagine a spacetime containing both big bangs (i.e. "lower" boundaries) and big crunches ("upper" boundaries), and such that the privileged "arrow of time" defined by the gradient of entropy in the vicinity of these various spacetime boundaries is, for each boundary, directed towards the interior of the spacetime (as it is for the boundary of our spacetime that is conventionally called "the Big Bang"). The simplest such spacetime, one with one big bang and one big crunch was suggested by Gold [13] as a model of our universe, and as an illustration of a conceivable correlation between the expansion of the universe and the increase of entropy. Hawking thought for a while that this time-symmetric model (featuring a reversal of the time-arrow around the stage of maximum size of the universe) might come out naturally from his Euclidean approach to quantum gravity [14]. However, Page [15] argued against this conclusion.

As already mentioned, we are considering that the "thermodynamic arrow of time", i.e. the direction of time with respect to which entropy grows, is what determines the sensation of "the passage of time", through the irreversibility of the process of memorization in the neuronal structures which give rise to the phenomenon of consciousness. In this view (which only assumes some minimal form of "psycho-physical parallelism") the "flow of time" is illusory, i.e. does not correspond to any "real" passage of time, while the "arrow of time" does correspond to a "real" structure of spacetime, namely a certain "stratification" of spacetime by hypersurfaces of varying entropy. [Note that this stratification is "static", and does not correspond to the common idea of a "stratum of the present" which would "move" towards the future, like a projector successively illuminating the various "entropy strata" of spacetime.]

Another example of a relativistic cosmos which puts into question the usual notion of temporal flow is the one introduced in 1949 by the famous mathematician Kurt Gödel [16]. Gödel's cosmos does not admit a "stratification" by global spacelike hypersurfaces. Locally, this spacetime admits a Lorentzian structure, i.e. it contains a regular field of lightcones separating timelike from spacelike directions. Near each point, one can therefore define pieces of spacelike hypersurfaces, and use them to distinguish the "upper" parts of the lightcones (the "future-directed" timelike vectors) from their "lower" parts (the "past-directed" ones). However, such a construction cannot be done globally because Gödel has shown that there exist "closed time-like curves" (CTCs), i.e. worldlines, representing the history of observers living in this cosmos, which close in on themselves like circles. In other words, in Gödel's spacetime it is possible to *travel into the past*. Gödel even showed that given any "starting" point P in spacetime (e.g. "here and now" for you, reader of those lines), and any wished "arrival" point Q (e.g. Mount Golgotha, on a certain Friday of April A.D. 33), one can travel (along an initially future-directed time-like path) from P to Q in a finite time (which can be, in principle, as short as wished). As far as we know, the structure of our cosmological spacetime does not include the feature of Gödel's one that leads to CTCs (namely the existence of a "rotation field" that can progressively tip the lightcones so as to reverse their orientation). However, the point of Gödel was not to claim that our universe is similar to his model but was to give a *conceivable* cosmos (solution of Einstein's field equations) in which the usual notion of universal time-flow becomes meaningless. The mere possibility of having such a solution[5] of Einstein's equations shows that, in General Relativity, the "external flow of time" can only be an "illusion", which depends on some particular structure of the spacetime we "live in". As is well-known, time travels can lead to paradoxical situations, but none of these paradoxes constitute a proof of non-existence. We should keep in mind, as an analogy, that the "twin paradox" has often been used as a proof of the inconsistency of the special relativistic time-dilation. We know, however, that it corresponds to a real effect, and that the "paradox" was just due to conceptual conservatism. For a detailed discussion of time travel's classical and quantum physics see [17].

6 Relativistic Gravity and the Second Law

In the last two Sections we have been mainly trying to show that, at the conceptual level, both the Special and the General Theories of Relativity suggest that one should open one's mind and stop being formatted by the traditional (and deeply ingrained) idea that Time exists as an entity outside the material world around us, and "drags" the "common now of the universe" as it "passes". In the words of Einstein :

"For us, physicists in the soul[6] the distinction between past, present and future is

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[5] It was later found that many other solutions of Einstein's theory of General Relativity can lead to time travel and CTCs : e.g. an overcritical rotating Kerr solution, solutions containing wormholes,...

[6] The German expression used by Einstein is "gläubige Physiker", which is often translated as "believing physicists". Nevertheless, all the philosophical context of Einstein's thought shows that one must not understand the word "believing" in the sense of a traditional religious belief, but rather in the sense of a deep belief in the rationality of the universe. Because of this, it seems to us more appropriate to translate "gläubige Physiker" by "physicists in the soul", or by "convinced physicists".

only a stubbornly persistent illusion."

In the following, we shall take for granted the idea that, as Einstein wrote once to his friend Michele Besso, "subjective time with its "now" [does] not have any objective significance", i.e. that it does not correspond to a "unique time, the same for all beings and for all things", as Bergson described the "common sense" idea of time. On the other hand, we shall take for granted that the subjective experience we have, as human beings, of the "flow of time" is ultimately rooted in the Second Law of thermodynamics, i.e. in the (objective) fact that, as said Boltzmann, "the universe, considered as a mechanical system – or at least a very large part of it which surrounds us – started from a very improbable state, and is still in an improbable state."

Within this view, the basic question that needs to be addressed is : what is the physical origin of the massive time-dissymmetry embodied in the very special past (and present) state of the entire visible universe ? This issue will be the main topic of the rest of this lecture. Let us start by noting that several different sectors of physics (or of the world around us) exhibit important time dissymmetries ;

1. Thermodynamics : the Second Law
2. Electrodynamics : retarded-potential radiation [7]
3. Expansion of the Universe
4. Irreversible behaviour of black holes (see below)
5. Quantum Mechanics : irreversibility in the Copenhagen interpretation of measurements.

Our view is that the facts 2, and the interpretation 5, have the same origin as 1, namely a very special state in the past (and today). We shall therefore focus on the points 3 and 4, i.e. on the question whether relativistic gravity (and cosmology) are related with the origin of the Second Law.

We saw above that Landau and Lifshitz suggested (starting in 1959) that relativistic cosmology might indeed be closely related with the Second Law. I am not sure who was the first to suggest such a connection. Though Friedmann [18] was the first to introduce time-dependence in cosmology, and to suggest a phoenix-type cyclic universe, undergoing successive bounces, I am not aware of his discussing the issue of the Second Law within such a model. This was specifically discussed by Tolman in 1932 [19]. Let us quote one of his main conclusions :

"The main purpose of this article has been a further examination of the bearings of relativistic thermodynamics on the well known problem of the entropy of the universe as a whole. The work has again illustrated the necessity of using relativistic rather than classical thermodynamics in treating this problem, and has demonstrated that the framework of general relativity at least provides a class of conceivable models of the universe which would undergo a continued series of expansions and contractions without being brought to rest by the irreversible processes which accompany these changes. The findings of relativistic thermodynamics thus stand in sharp contrast to the familiar conclusion of the classical thermodynamics that the continued occurrence of irreversible processes would lead to an ultimate condition of maximum entropy and minimum free energy where change would cease."

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[7] Note that the observations of binary pulsars have also shown that gravitational radiation is emitted via *retarded* potentials, rather than advanced ones.

The point of Tolman is interesting but does not really address the issue of the origin of the Second Law.

A few months before the paper of Tolman (which was submitted on November 13, 1931) the issue of time asymmetry (and "time's arrow") in cosmology was discussed by Eddington and by Lemaître. See the lecture of Huw Price for a discussion of Eddington's ideas. Here, I will only consider the ideas of Lemaître, focussing on his remarkable Letter to Nature, Ref. [20]. This Letter is very short and is worth quoting in its entirety :

"**The Beginning of the World from the Point of View of Quantum Theory.**

Sir Arthur Eddington[8] states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical principles from the point of view of quantum theory may be stated as follows : (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed in a few or even in a unique quantum.

Now, in atomic processes, the notions of space and time are no more than statistical notions ; they fade out when applied to individual phenomena involving but a small number of quanta. If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning ; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

It may be difficult to follow up the idea in detail as we are not yet able to count the quantum packets in every case. For example, it may be that an atomic nucleus must be counted as a unique quantum, the atomic number acting as a kind of quantum number. If the future development of quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radio-active process. Some remnant of this process might, according to Sir James Jean's idea, foster the heat of the stars until our low atomic number atoms allowed life to be possible.

Clearly the initial quantum could not conceal in itself the whole course of evolution ; but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens ; the whole story of the world need not have been written down in the first quantum like a song on the disc of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step."

Though the formulation of Lemaître is somewhat unclear and imprecise, it carries a deep vision of a possible interlocking between : (1) the Second Law ; (2) Quantum Mechanics ; and (3) the emergence of the universe (and of space and time)

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[8] *Nature*, Mar. 21, p. 447.

from "a single quantum" [that he later explicitly connected to his work on expanding cosmological models, with a cosmological constant, under the name of "the primeval atom" ("l'atome primitif")]. Besides the vision, what can be retained today of the suggestions of Lemaître is unclear. However, the issues discussed by Lemaître have been hotly discussed ever since. The already mentioned works of Gold, of Hawking and of Page, being examples of such discussions.

Let us note that both Tolman and Lemaître (especially in his subsequent, more detailed papers) were mainly discussing the issue of the increase of the entropy of the material content of the universe, taking the Einsteinian spacetime essentially as an external, self-consistent time-dependent background. The issue of the "entropy" to be attributed to this external gravitational field was (apparently) not considered. This issue got a decisive impetus from the work of Christodoulou [21], Christodoulou and Ruffini [22], and Hawking [23] on the *irreversible* aspects of the physics of black holes. These authors discovered that, during the interaction of one or several black holes with external particles or fields, a certain quantity (the square irreducible mass, or the area) could only increase. The later work of Bekenstein [24] and Hawking [25] led to attribute to any black hole the entropy (with $c = 1$)

$$S_{\mathrm{BH}} = \frac{A}{4\,G\hbar} \tag{1}$$

where A is the area of the horizon.

The statistical physics meaning of Eq. (1) is still rather mysterious (in spite of remarkable results in string theory), but there is no doubt that it is telling us something deep about a three-way link between quantum theory, general relativity and thermodynamics.

7 Primordial cosmology and the Second Law

To end this brief survey, let us mention some of the recent attempts at connecting the Second Law with primordial cosmology. Several possibilities have been suggested (references on the recent works alluded below can be easily obtained from the web, or from the books quoted at the beginning of this lecture)

- The "chaotic inflation paradigm" (Linde) [or, alternatively, the eternal inflation paradigm (Vilenkin, Linde)] argues that our entire visible universe developed from a roughly homogeneous Planck-scale patch of a "random" universe. The inflationary mechanism, together with the a posteriori condition of looking only at large (inflated) patches, seems to naturally introduce a dissymmetry, explaining why the post-inflationary universe "starts" in a rather low-entropy state (compared, say, to the present one)
- The "special boundary paradigms" wish to add to the dynamical laws of nature, an additional prescription to select the global state of the universe. For instance, R. Penrose suggests to impose the vanishing of the Weyl curvature at "initial" spacetime singularities. Another example, is the "no-boundary" proposal of Hartle and Hawking which tries to restrict the quantum amplitude of the universe by generalizing the Euclidean-time characterization of "ground state" wavefunctions in quantum field theory.

- The “quantum tunnelling paradigms” wish to describe our universe as the result of a quantum tunnelling from some previous state. [Note that this is reminiscent of the “super-radioactive process” contemplated by Lemaître.] The “tunnelling from nothing” scenario of Vilenkin is similar to the Euclidean-time description of pair creation. Many scenarios explored various possible tunnellings between different “vacua” of some underlying theory (Garriga and Vilenkin, Dyson, Kleban and Susskind, Albrecht and Sorbo, Carroll and Chen, ...).

All these studies have usefully stretched our imagination about the possible origin of our world. However, it is not clear that any of them provides a satisfactory answer to the basic question of the origin of the Second Law. For instance, the chaotic inflation scenario looks a priori quite appealing. It uses the “ironing” effect of inflation to stretch a small, inhomogeneous patch into a huge, nearly homogeneous space. Moreover, as the inflationary behaviour is a dynamical attractor, some authors have argued that “most” initial states will be inflated and thereby ironed out (Belinsky, Khalatnikov, Grishchuk and Zeldovich 1985, Kofman, Linde and Mukhanov 2002). But, other authors (Khalfin 1989, Hollands and Wald 2002) have argued that “very “special” initial conditions are nevertheless needed in order to enter an era of inflation”. Basically, their argument is similar to the old objections of Kelvin, Maxwell, Loschmidt and Zermelo to Boltzmann : the time-reversibility of the underlying (general relativistic) dynamics, and the invariance of some Liouville measure imply that *any* present state of the universe (as inhomogeneous as wished), must come from *some* initial state. Therefore the latter initial state was *not* ironed out by inflation, which shows that only a special class of initial states can be ironed out by inflation. In spite of the apparent strength of this argument, some specific aspects of gravity, and of the interplay between gravity and quantum mechanics, make it problematic. We have here in mind three issues :

- Contrary to what happens in usual dynamical systems, or for usual (non gravitational) fields, the Liouville measure of spatially compact, finite-energy systems *does not have a finite integral* when one includes the gravitational degrees of freedom. [This comes from a famous *minus sign* associated to the conformal mode in gravity.] Because of this, we cannot use the Liouville measure (or its various possible quotients) to estimate the likelihood of some state.
- Some authors (most notably R. Penrose) estimate the probability of the initial state by assuming that the irreversible behaviour linked to gravitational clumping in our universe can be quantified in terms of some “entropy” S_g linked to the gravitational field ; and they use the Bekenstein-Hawking Black Hole entropy (1) to estimate S_g now. However, this type of estimate is not justified by our (rather incomplete) knowledge of the thermodynamics of self-gravitating systems.
- Quantum gravity considerations suggest that inhomogeneous modes having Planck-scale wavelengths ($\lambda \sim \ell_P \equiv \sqrt{\hbar G}$) must be (effectively) excluded from the Hilbert space of physical quantum states. This has two types of effects : (1) it introduces a high-frequency cut-off and thereby allows inflation to iron out *all* the initial states with $\lambda \gtrsim \ell_P$; (2) it effectively introduces a violation of the conservation of the number of states (which is the quantum version of Liouville's theorem) during the expansion.

In conclusion, we see that the basic issue raised by Boltzmann long ago is still with us. What is new is that we now think that its answer (if any) lies at the interplay between relativistic gravity and quantum mechanics. This reminds us of the suggestion of Landau and Lifshitz, except that most people interested in this issue do not interpret quantum mechanics in the Copenhagen way, but rather in the Everett's one [26].

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